Wage-Profit Curves in a von Neumann-Leontief Model: Theory and Computation of Japan's Economy 1970 – 1980[1]

By
Yoriaki Fujimori*

Abstract

Objectives of this paper are (i) to estimate marginal capital coefficients of the Japanese economy; (ii) to draw the wage-profit curve of the Japanese economy, and (iii) to find some facts concerning the dynamic structural changes of Japan from 1970 to 1980. The framework of the analysis is a von Neumann-Leontief model that permits the existence of aged fixed capital, and the basis for computation is the input-output table with 35 industries.

Our analysis yields three results; (i) the wage-profit curve of Japan's economy is of approximately linear form; (ii) the wage-profit curve moved west-southward in the analysis period — this fact corresponds to the increase in utilisation of resources owing to the so-called oil crisis, the necessity to reduce pollutants and to control industrial wastes, research-and-development investment and the high-technology-oriented strategy of firms; (iii) the dynamic classification of industries reveals some slight changes.

1. Introduction

Some dynamic features of the economy are based on a single production period, because a basis of growth is the production of each period. One of them is revealed by the investment-consumption frontier.

The investment-consumption frontier has been one of the most important topics in economic theory. As opposed to its theoretical analysis, however, empirical analysis of the wage-profit curve has not been made to its fullest extent. We do not know, in fact, how the actual wage-profit curve looks. The reason why the wage-profit curve of the actual economy has not been computed may be reduced to two difficulties related to fixed capital: firstly, the capital coefficient matrix was not necessarily available, and secondly, the computational method to evaluate the wage-profit curve in a von Neumann system has been considered very complicated.[2]

To draw the wage-profit curve will enable us to solve some problems. The first problem is the information concerning potential growth of the actual economy. The other is which sector is most likely to increase its weight of equilibrium output and/or price of a referred economic system as consumption tends to vanish: this might be called the dynamic classification of sectors.

We shall study these problems both in the short-run and the long-run position. By short-run, we mean that the replacement and net investment of fixed capital are negligible, while the long-run indicates that they matter.

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*School of Political Science and Economics, Waseda University
[1]The author acknowledges thanks to those who commented on an earlier draft read at the International Conference on Input-Output Techniques, July 28-August 2, 1986, Hokkaido. He also owes to Professor Dr. J. Tsukui (Osaka Univ.) for his helpful comments. Needless to say, any remaining errors should be blamed on the author.
[2]Tsukui-Murakami[13] is the first systematic application of the turnpike theory to empirical analyses, with the estimation of capital coefficients for the Japanese economy of 1965.

2. Wage-Profit Curves in a von Neumann-Leontief System

Ordinary production processes of today employ fixed capital of various durabilities and different ages: each process produces, strictly speaking, aged fixed capital as well as brand-new goods.

Let us suppose an economy in which aged and brand-new goods are distinguished. An economy looked at as such will be called a *raw* economy. One can describe the equilibrium of a raw economy *á la* von Neumann as: for a given $g \in \boldsymbol{R}$,

$$\min\{Lx \mid \frac{1}{1+g} Bx \geq Ax + F, x \geq 0\}, \tag{1}$$

and for a given $r \in \boldsymbol{R}$,

$$\max\{pF \mid \frac{1}{1+r} pB \leq pA + L, p \geq 0\}, \tag{2}$$

where A is an input-matrix, B an output-matrix, F a wage-good bundle and L a labour vector of the raw economy respectively; x and p are vectors of relevant dimensions. If $r=g$, these two constitute dual problems to each other.

As we showed elsewhere, however, equilibria of this joint-production system *á la* von Neumann can be reduced to those of a Leontief system which does not contain any aged fixed capital, if fixed capital can be operated with the same efficiency irrespective of its ages. If we assume that the same type of fixed capital can be regarded as undistiguished irrespective of its ages, the representation of techniques of the system can be made only in terms of brand-new goods. A technique will be called *basic*, when it is expressed in terms of brand-new goods.

Let us consider a Leontief economy with m sectors producing m types of goods. We shall employ the following notations concerning the basic techniques:

A	$m \times m$	: current input matrix,
K	$m \times m$	: fixed capital input matrix,
L	$1 \times m$	: labour vector,
F	$m \times 1$	: consumption basket vector,
τ_i		: durability of fixed capital i,
ψ_i		: rate of depreciation of fixed capital i.

In order to evaluate the wage-profit curve as indicated by the relation between the profit rate and the number of units of the consumption goods bundle, let us apply, as suggested above, the linear programming approach to the economy with fixed capital discussed here.

One can consider

$$\min\{Lx \mid \frac{1}{1+r} x \geq Ax + F + (\frac{r}{1+r} I + \frac{1}{1+r} \hat{\psi}(r))Kx,\ x \geq 0^m\}, \tag{3}$$

in lieu of (1), or,

$$\max\{pF \mid \frac{1}{1+r} p \leq pA + L + p(\frac{r}{1+r} I + \frac{1}{1+r} \hat{\psi}(r))K,\ p \geq 0_m\}, \tag{4}$$

in lieu of (2), where $\hat{\psi}$ is a diagonal matrix, the ith element of which is

$$\psi_i(r) = \frac{r}{(1+r)^{\tau_i} - 1}.^{3} \tag{5}$$

[3] If the economy has been in balanced growth with no capitalist consumption, the same depreciation rate can be applied to the quantity system. See Fujimori[2], Ch. 2.

The wage-profit curve is represented by a graph $(r,h) \in \boldsymbol{R}^2$, with $h = \dfrac{1}{p^*F} = h(r)$, where p^* is an optimum solution of (4). Since (1) and (2) are dual, they give the identical frontier.

Hence, (3) and (4) yield the same wage-profit curve, $h(r)$. Therefore, in what follows, we mention the (equilibrium) profit rate and the (equilibrium) growth rate interchangeably. For the sake of computational convenience, (2), and hence (4), is regarded as primal in the following.

It is worth mentioning that on the basis of the von Neumann system, the Leontief system can be looked at from a new and widened angle.[4]

The last term of the above (dynamic) quantity system (3) represents two portions of gross investment of fixed capital, i.e., net investment and replacement of fixed capital. For the convenience of discussions which will be made later, let us make clear some relationships between gross investment and net investment of fixed capital. Let us write:

v_i: ratio of net investment to gross investment of fixed capital i

and one has

$$v_i = 1 - \frac{1}{(1+r)^{\tau_i}}. \tag{6}$$

3. Data and the Estimation of Marginal Capital Coefficients

3.1. Empirical Basis

So far we have reviewed some theoretical background for the wage-profit curve. Our next task is to procure necessary data for our computations.

The necessary matrix and/or vector data for our purpose are as follows: a current input coefficient matrix A, a labour vector L, a consumption basket F, durability of fixed capital τ, and a capital coefficient matrix K.

In what follows, we shall consider some formulae which present a basis to estimate various types of coefficients employed in our computations.

The fundamental equation system of the actual input-output table is:

$$\tilde{x}_j = \sum \tilde{x}_{ij} + \tilde{Y}_j, \tag{7}$$

$$\tilde{Y}_j = \tilde{C}_j + \tilde{\Phi}_j + \tilde{y}_j + U_j, \tag{8}$$

$$\tilde{N}_0 = \sum_{j=1}^{n} \tilde{N}_j, \tag{9}$$

[4]In fact, the usual dynamic Leontief system is described by $x = Ax + B\dot{x}$, but in this system replacement and net investment of fixed capital are regarded as independent of growth rates. This is due to the fact that aged fixed capital is not treated properly in the Leontief system. As for the theoretical features of the Leontief system, see e.g. Fujimori [82].

A straightforward inclusion of fixed capital into the Leontief system will yield the production price system of the economy $(p,r) \in \boldsymbol{R}^n \times \boldsymbol{R}$ expressed by

(*1) $$p = (1+r)(pA + pcFL) + p[rI + \hat{\psi}(r)]K,$$

and the balanced growth path of the economy $(x,g) \in \boldsymbol{R}^n \times \boldsymbol{R}$ without capitalist consumption represented by

(*2) $$x = (1+g)(Ax + cFLx) + [gI + \hat{\psi}(g)]Kx,$$

where c indicates units of consumption goods bundle. As seen from these, to evaluate r or g as a function of c is not easy, because their relationship is fundamentally nonlinear.

The relationships between growth and consumption or those between wages and profits as based on (3),(4),(*1) and (*2) are substantively equivalent. For example, the linear programming problem (4) means that the price of a consumption bundle is maximised with constant nominal wages being unity. The reciprocal of pF thus indicates real wages — how many units of a consumption bundle can be purchased by a unit of nominal wages. As for the details, see Fujimoto [75].

where

$\tilde{x}_j$: total domestic production of sector j,
$\tilde{x}_{ij}$: current inputs from sector i to j,
$\tilde{Y}_j$: final demand of sector j,
$\tilde{C}_j$: private consumption of good j,
$\tilde{\Phi}_j$: gross investment of fixed capital j,
$\tilde{y}_j$: final demand of sector j other than private consumption and gross investment of fixed capital,
U_j : export-import difference of good j,
$\tilde{N}_0$: total number of persons engaged,
$\tilde{N}_j$: the number of persons engaged in sector j,

with $i,j = 1,\ldots,n$; n, the number of sectors.

From the above data, $A = (a_{ij})$, $L = (L_j)$ and $F = (F_j)$ are estimated in the conventional manner:

$$a_{ij} = \frac{\tilde{x}_{ij}}{\tilde{x}_j}, \tag{10}$$

$$L_j = \frac{\tilde{N}_j}{\tilde{x}_j}, \tag{11}$$

$$F_j = \frac{\tilde{C}_j}{\tilde{N}_0}. \tag{12}$$

Remark that self-employed and executives are included in the category of the worker here. Some may argue that they are not workers and hence should be excluded, but in reality their participation in production lines and other fields of businesses is indispensable in considering the potentiality of the actual economy. In fact, most self-employed workers in agriculture are performing some kind of production activities, whereas some executives play an important role with reference to information activities of firms. Therefore, labour vectors are estimated on the engaged-worker-base in our computation.

Also remark that consumption goods purchased by non-workers are also included in our computation. This means that the bundle of wage goods estimated in our computation may involve luxury goods to some extent.

The method to estimate capital coefficients depends on available data concerning fixed capital. Our estimation is to make use of the matrix of gross investment of fixed capital.

The marginal capital coefficient is evaluated by

$$k_{ij} = \frac{v_i \tilde{\Phi}_{ij}}{g \tilde{x}_j}, \tag{13}$$

where

k_{ij} : marginal capital coefficient,
$\tilde{\Phi}_{ij}$: gross investment of fixed capital i to sector j,
g : growth rate.

Remark that ν_i here depends on g.

This method, however, raises another question. We should know the relevant potential growth rate, g, of the year concerned in order to evaluate the appropriate increments of production of sectors.

One may argue that investment of fixed capital depends on expectations concerning the future and hence the relevant growth rate cannot be read from input-output tables, but we assume further that the growth rate derived from the input-output data coincides with the expected growth rate.

3.2. Basis of Simulation

As discussed above, the amount of net investment of fixed capital in general depends on the rate of growth. It must be observed, however, that the capital coefficients will remain the same even when the economy is growing at its greatest rate with zero wages, in so far as techniques are linear and no technical changes occur. Though the actual economy is not in balanced growth and a part of products is consumed, one can concentrate on the estimation of capital coefficients, if the growth path with the greatest growth rate can ever be simulated, because in that situation all products are invested. This implies that we have to simulate the standard commodity *á la* Sraffa.

The basic procedure can be outlined as follows. Suppose that we have a 1-sector-1-good closed economy

$$X = AX + Y, \tag{14}$$

where

$$Y = I + C, \tag{15}$$

I indicating investment, C consumption. (These two correspond to (7) and (8) respectively.) Assume that the economy is in maximum growth with zero wages at R^*. Then, R^*AX must be alloted for increments of nondurable capital, so that

$$q\ (R^*) = Y - R^*AX$$

can be alloted for investments of fixed capital by assuming Y/AX (total net product/total current input) is constant. Suppose that the durability of fixed capital τ is known. Then, $K(R^*) = \nu(R^*)$ $q(R^*)/R^*X$ gives a marginal capital coefficient. Since this R^* is the greatest growth rate, the desired magnitude of R^* is the smallest one among those which satisfy (4) with $P(R^*)\ F \rightarrow \infty$.[5]

3.3. The Greatest Profit Rates

Our problem here is fivefold. The first point is how to evaluate the available amount of net products out of the actual final demand data which consists of various factors.

At the outset, when we say the total net product is invested in the standard system, this total net product corresponds to the sum of consumption and investment of the input-output tables.

In the next place, we shall adjust the ratio of the total net product to the total current input. In general, this ratio depends on the growth rate. We shall assume that the amount of governmental consumption should be subtracted from the total net product. This is because luxury goods are not produced in maximum growth.

The third is how to apply the above mentioned basic idea to multi-sectoral input-output systems. The system in which we evaluate the greatest growth rate to define marginal capital coefficients is the standard system, so that pure consumption goods are not produced at all therein. This fact creates a difficulty in finding the multipliers for each sector and for each type of good. Hence, we shall employ an overall ratio to augment gross investment of fixed capital.

The fourth point is the range of fixed capital. In the compilation of gross investment matrices,

[5]The actual computation involves difficulty for two reasons. Firstly, A is not necessarily nonnegative, because some types of byproducts are treated by the so-called Stone method, and, secondly, K and ψ (g) vary in the course of the computation.
It is worth mentioning, however, that despite the fact that some elements of A are negative, the iteration proposed here reveals a global convergence because conditions specified in the above are satisfied in our computation.

Table 1 Fundamental Macro Parameters

	1970		1980	
	case (1)	case (2)	case (1)	case (2)
maximum growth rate (%)	48.04	30.00	45.16	35.15
macro marginal capital/output ratio	0.589	2.939	0.628	1.757

fixed capital is divided into two categories: one is production capital, and the other is public capital. The range of fixed capital matters in considering maximum growth. Hence, with regard to the treatment of public capital, we will distinguish two cases:

(1) The case in which public capital is disregarded from the economy.

(2) The case in which public capital is treated as part of production capital laid out in advance in proportion to production levels.

In case (1), the amount of investment of public capital is subtracted from the available net products. In case (2), on the other hand, the amount of investment of public capital is regarded as part of production capital, and hence it immediately affects marginal capital coefficients. Thus, we have two types of estimation of marginal capital coefficients, i.e., case (1) and case (2).

The final, fifth point is: as might be expected from our basic idea to estimate marginal capital coefficients, our computation here is an iterative procedure to find the maximum growth rate. We shall assume that the economy will remain in the same position in the following sense that the macroeconomic situations, such as imports and exports etc., will not be affected, even if rates of profit change.

Sources of data for our simulation are input-output tables of 1970 and 1980 compiled by the Government of Japan [4, 5]. Our present computation is made on the basis of the 35-sector input-output system. As for the list of 35 sectors in our computation, refer to Appendix A. Observe that input-output tables of 1970 and 1980 are compiled on similar code-systems, but their contents are slightly different from each other. Hence, some necessary modifications will be made in the following computation. As for the remarks concerning the modifications of data, see Appendix B.

We should mention that figures to indicate durability of fixed capital are borrowed from Tsukui [11]. As for those figures, refer to Appendix A.

Table 1 shows the magnitudes of growth rates, at which marginal capital coefficients are estimated, and the macro marginal capital-output ratios of both 1970 and 1980, for cases (1) and (2). (Remark that those figures are approximated ones.)

4. Computations of Wage-Profit Curves for Japan

4.1. Wage-Profit Curves

Heretofore, we have procured all the necessary data, so that we can draw wage-profit curves of the Japanese economy for 1970 and 1980. In both years, we estimated short-run and long-run classifications of sectors with respect to the von Neumann equilibrium.

Our second and main computation is carried out by solving (4) and/or (3) for given magnitudes of the profit rate. Table 2 gives the real wage level $1/pF$ for equilibrium profit rates π. The magnitude of steps of the profit rate is 0.05.

Figure 1 and Figure 2 show the short-run graph of (π, $1/pF$) and the long-run graphs of both cases (1) and (2), for the years 1970 and 1980,

Table 2 Wage-Profit Function

π	1/pF					
	1970			1980		
	short-run	case (1)	case (2)	short-run	case (1)	case (2)
0.00	1.759	1.713	1.627	1.686	1.628	1.578
0.05	1.596	1.541	1.418	1.522	1.452	1.376
0.10	1.448	1.354	1.143	1.370	1.253	1.128
0.15	1.312	1.181	0.879	1.229	1.069	0.896
0.20	1.187	1.018	0.613	1.097	0.894	0.675
0.25	1.070	0.861	0.328	0.973	0.726	0.458
0.30	0.962	0.706		0.853	0.559	0.238
0.35	0.859	0.546		0.737	0.389	0.004
0.40	0.761	0.371		0.622	0.208	
0.45	0.665	0.161		0.504	0.004	
0.50	0.569			0.381		
0.55	0.466			0.242		
0.60	0.343			0.074		
0.65	0.154					

Note: Blanks indicate zero.

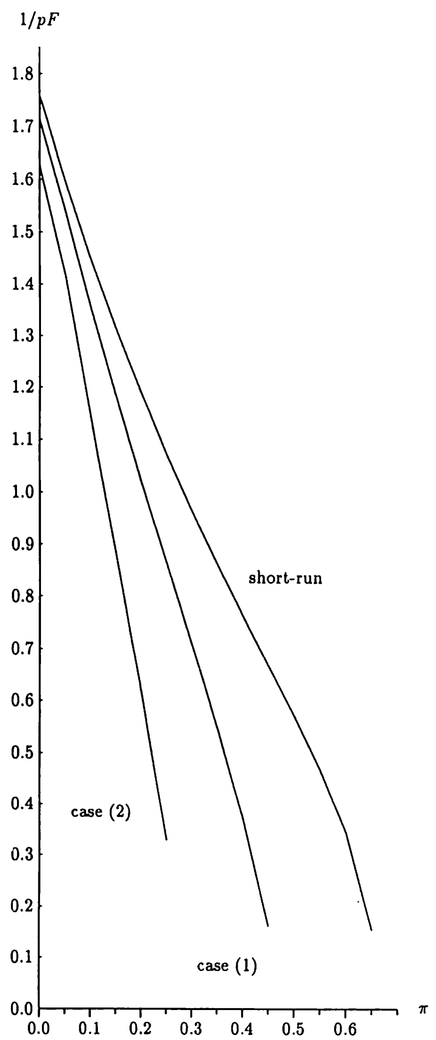

Figure 1 1970 Wage-Profit Curve

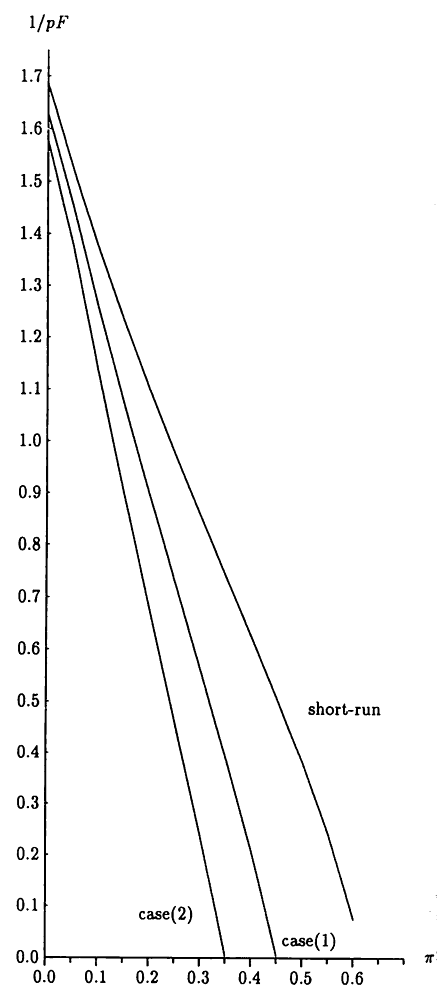

Figure 2 1980 Wage-Profit Curve

Table 3 1970-1980 Short-Run

	1970	1980
Pro-growth sectors	11, 13, 14, 16, 22, 33, 34, 35 (8,10,12,15,18)	4, 10, 11, 12, 13, 14, 15, 16, 18, 22, 29, 33, 35 (8,17,34)
Neutral sectors	2, 4, 7, 17, 19, 21, 27, 28 (31)	2, 7, 26, 28
Pro-consumption sectors	(20, 23, 30) 1, 3, 5, 6, 9, 24, 25, 26, 29, 32	(9, 19, 20, 23, 27, 31) 1, 3, 5, 6, 21, 24, 25, 30, 32

Table 4 1970-1980 Long-Run

	1970	1980
Pro-growth sectors	11, 13, 14, 15, 16, 17, 18, 22, 33 (4,12,34)	2, 4, 11, 12, 13, 14, 15, 16, 17, 18, 22, 33, 35
Neutral sectors	(8, 21, 28, 35) 2, 7, 10, 20, 23, 24, 27, 30 (29, 30)	(10, 29) 7, 21, 23, 24, 27, 34 (8, 28, 30)
Pro-consumption sectors	(19) 1, 3, 5, 6, 9, 25, 26, 32	(9, 20) 1, 3, 5, 6, 19, 25, 26, 31, 32

4.2. Dynamic Classification of Sectors

Let us turn to the optimum solutions of (4) and its dual. Let

$p_i^*(\pi)$: optimum price of good i,

$x_i^*(\pi)$: optimum output of sector i,

and we have

$$p_i^{**}(\pi) = \frac{p_i^*(\pi)}{\sum p_i^*(\pi)},$$

$$x_i^{**}(\pi) = \frac{x_i^*(\pi)}{\sum x_i^*(\pi)}.$$

We look at those proportions of prices and outputs as a function of equilibrium profit or growth rates. The problem is whether or not those functions reveal some specific characteristics.

Some of those functions, p_i^{**} and x_i^{**}, are monotone functions. In view of this fact, we can classify sectors from the dynamic standpoint. Sectors with increasing output and price proportions are regarded as a *pro-growth* sector, those with decreasing proportions as a *pro-consumption* sector, and those which go between them as *neutral*.

The subsequent tables, Table 3 and Table 4, summarize the classification: figures in the tables indicate sector-numbers. Those in ( ) are near the border.[6]

As for details of the above tables, refer to Appendix C.

4.3. Some Remarks

It will be appropriate to make some observations from two angles. One is to compare 1970 and 1980, and the other is to compare the short-run and the long-run.

[6]Remark that p^{**} and x^{**} of the public service sector take zero for 1970.

The wage-profit curve indicates the potentiality of the economy in general, but the potentiality of the Japanese economy was not necessarily increased in the period concerned. This may be explained by several reasons. Firstly, the so-called oil crisis hit the world economy in that period; secondly, it became necessary to increase the amount of investment for the reduction of pollutants and industrial wastes; thirdly, the research-and-development activities started to absorb more investment; fourthly, the strategy of firms to supply more high-technology-oriented products required additional spendings. All those additional costs have not come to contribute to an increase in the growth potentiality of the Japanese economy in the period concerned.

It must be observed that there is a cross of wage-profit curves of 1970 and 1980 in case (2). This implies that the shift of the potentiality of the economy as indicated by the wage-profit curve is not uniform from 1970 to 1980.

By looking at wage-profit curves, some facts are made clear. We immediately observe that those curves are not necessarily linear, although some of the long-run wage-profit curves seem to be loosely approximated by a line.[7] This suggests that one may well introduce a linear wage-profit curve in the first step of analysis of macroscopic distribution models. Moreover, it is not difficult to see that the long-run wage-profit curve is steeper than the short-run wage-profit curve. This is one of the effects of fixed capital. The higher the profit-rate is, the more difficult it becomes for the economy with fixed capital to grow.

It must be remarked that $\frac{1}{pF}-1$ where $\pi=0$ indicates the so-called rate of surplus value.[8] As far as our estimation shows, the rate of surplus value decreased from 1970 to 1980.

Let us next deal with the dynamic classification of sectors.

We observe that some sectors reveal a very clear identification both in the short-run and the long-run. Agriculture (sector 1), for example, is a pro-consumption sector, while the iron and steel industry (sector 13) is a complete pro-growth sector.

If we compare the short-run and the long-run more carefully, however, we observe that a few sectors change their places from pro-growth to neutral or from pro-consumption to neutral sectors and vice-versa. Hence, from the comparison between the short-run and the long-run, we can say that there is a meaningful difference between the two.

From the standpoint of the dynamic classification, structural changes which arose in the Japanese economy 1970-1980 are not large in the sense that major sectors remain in the same category, although, from 1970 to 1980, the number of sectors belonging to the pro-growth and the pro-consumption sectors increased. This implies that the classification of sectors in 1980 becomes much clearer than in 1970.

5. Postscript

So far, by using the linear programming approach, we made an attempt to apply the Marx-Sraffa-type linear economic theory to the wage-profit curve for the Japanese economy.

In the above, we have pointed out some theoretical assumptions and actual restrictions related to available data themselves.

The actual input-output tables are compiled on the basis of market prices of products. This limits the range of validity of analysis of this kind. To use input coefficient matrices on the market price basis, even if the idea of a dollar's worth is applied, means the following: the whole analysis

[7] Tsukui[11] observed a linear relationship between consumption and growth.

[8] Ekaizu-Kuroiwa[1] evaluated the rate of surplus value for the Japanese economy by estimating the wage bundle from the survey of the household. Their magnitude (from 15% to 30%), however, seems to be greatly underestimated.

concerned is valid on the assumption that the market situation will never be affected by, say, the growth rate.

Nevertheless, we can say that there is a possibility to estimate marginal capital coefficients, provided that matrices of investment of fixed capital are available.

Now, the wage-profit curve as discussed so far may represent the potentiality of the state of the Japanese economy fully equipped by the latest techniques as indicated by marginal fixed-capital coefficients. To draw the wage-profit curve will provide us with the possibility that a normative level of the balanced growth rate is obtained, in case the balanced rate of savings is known.

The precision of the whole computation, however, depends on the estimation of marginal capital coefficients.

From the standpoint of the precision, the wage-profit curve thus obtained may leave much to be desired. The information provided by the wage-profit curve will be of qualitative nature, such as linearity, an intertemporal shift of the potentiality and the existence of a cross of wage-profit curves of 1970 and 1980 in case (2).

We have to say, moreover, that marginal capital coefficients are rather underestimated, because the marginal capital-output ratios are too small. This reveals that there is a discrepancy between the actual state of the economy and the standard system referred to. It will be possible, however, to improve the estimation in connection with other statistical data.

Yet, considering all those restrictions with regard to data, such an analysis as we have made will provide important information concerning some of the dynamic features of the economy which are created by, in particular, the presence of fixed capital.

Table 5 Codes of 35 Goods/Sectors

Code	Goods/Sectors	Durabilities (years)
1	Agriculture	14.0
2	Forestry	
3	Fishery	
4	Mining	
5	Foods	
6	Textile	9.0
7	Woods and woodwork	9.0
8	Pulp and papers	
9	Leather and rubber	
10	Chemicals	
11	Petroleum and coal	
12	Ceramics	
13	Iron (and steel)	
14	Non-iron metals	9.0
15	Metal works	9.0
16	General machinery	15.0
17	Electric machinery	15.0
18	Transportation machinery	6.4
19	Precise machinery	15.0
20	Other manufactured goods	9.0
21	Construction	35.0
22	Power supply	
23	Gas	
24	Water	
25	Commercial	9.9
26	Finance and insurance	
27	Real estate	
28	Real estate rental	
29	Transportation	12.4
30	Communication	
31	Public services	
32	Other services	
33	Business supplies	
34	Packing	
35	Unclassified	

Appendix

A. Goods/Sector Code

Our computation is based on the 35-sector input-output table. Its code system is shown by Table 5.

Blanks in the durability column indicate that goods concerned are not durable: they are consumed in one year.

B. Data Modification

This appendix describes the remarks and the modifications of data made in preparation for our computation.

The 35-sector input-output system comes from the fixed capital matrix data which is published in the 84×34 or 94×34 matrix format. Our code system is based on the 34-production-capital-functions of the fixed capital matrix table. In constructing the 35-sector system, however, the packing sector is added to the original 34-sector system.

Although fundamental matrices of intermediate transactions larger than 500×400 are available, current input coefficients employed in our simulation are based on aggregated tables from 60×60 (1970) to 72×72 (1980) cross-section data. As mentioned above, these data are aggregated to 35-sector tables.

The amounts of aggregated gross investment of fixed capital are shown in the fixed-capital-formation column in input-output tables, but since 1970 the matrix data of gross investment of fixed capital has been compiled together with input-output tables. The sum of the gross investment matrix is not equal to the figure in original input-output tables, so that gross investments are modified in order for their sum to be equal to the figure in the input-output table.

The amount of public capital comes mostly from the construction sector. Hence, when computing gross investment of case (2), the amount of public capital is added to each sector in proportion to the investments from the construction sector to each sector.

The fixed capital matrix of 1970 is compiled on a slightly different basis from that of 1980. Hence, we modified the fixed capital matrix of 1970 in the following manner: the public investment for governmental buildings is added to productive capital investment for the public sector. (Remark that the public investment for governmental buildings does not exist in the year 1980.)

As for the public service sector, note that in the original compilation of input-output tables this sector is treated differently in 1970 and 1980.

C. Classification of Sectors

Classification of sectors is made in the following manner.

We find linear expressions to connect two points, $(0, s_i^{**}(0))$ and $(\delta^*, s_i^{**}(\delta^*))$, where $s_i^{**}(\ )$ indicates either $x_i^{**}(\ )$ or $p_i^{**}(\ )$. On this line, we evaluate $\gamma_i = 1 + \left(\frac{s_i^{**}(1)}{s_i^{**}(0)} - 1\right)\frac{1}{R*}$ as an index of the slope of the line. If $\gamma_i < 0.8$, the function is classified as decreasing; if $\gamma_i > 1.2$, then the function is regarded as increasing; otherwise, it is neutral. The combination of price and output proportions enables us to classify sectors into nine groups. Sectors with the combination of increasing output and increasing price proportions are regarded as pro-growth sectors, while those with decreasing ones are regarded as pro-consumption sectors. The remaining ones are classified as neutral.

It is worth mentioning that some of those $s_i^{**}(\pi)$ reveal monotonicity. Needless to say, this observation depends on the step of the profit rate of our computation, but it should be remarked that industries which can be clearly classified as pro-growth or pro-consumption industries are likely to show the monotonicity. The magnitude of steps of the profit rate for the short-run computation is 0.05, while that for the long-run is 0.02.

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