Input-Output Marketing Analysis Table: Consumer Behavior Analysis Based on Closed Systems[1]

By

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Abstract

The current analysis model calculates the transition probability matrix based on the results of market research of consumer behavior. This analysis is introduced to solve the problem of estimating consumer behavior and to forecast the consumer behavior among three trade areas. Furthermore, consideration based on the results of market research is given to this theoretical model. The significant features of the analysis model are the transition probability matrix among trade areas based on a sample of inflow-outflow investigation for consumer behavior, and the attraction index of trade area introduced from the transition probability matrix.

1. Introduction

We propose the analysis theory based on market research to forecast consumer hehavior among regions. Consumer behavior is dependent on the attraction of retail supply as well as the real distance between the physically separated demand and supply. It has been argued by several authors that consumer behavior can be related to physical position between consumer demand and retail supply (Reily, 1929; Converse, 1949; Kotlar, 1971). Also, a network model for consumer behavior has been studied from an analogy of classical transport in resistor networks (White and Ellis, 1971). Much significant study has been conducted in this field. Among many other advances, Huff has conducted many practical experiments on consumer behavior and developed a detailed analysis theory (Huff, 1962).

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There is, however, no analysis theory for consumer behavior based on market research. In this paper, we propose a probability analysis model that uses information obtained from market research. The conventional studies that have used only physical factors could not realistically forecast dizzily changing consumer behavior. We must carefully inspect and analyze the structure of consumer behavior, and must reflect the information obtained through market research in the consumer behavior model. This analysis does not depict the relationship between consumers and retail stores, but it does analyze the relationship between a region with stores and its competing regions.

In Section 2 we describe the method of constructing the fundamental model which analyzes consumer behavior of trade areas in a city. Next we present in theory the forecast of consumer behavior when the number of stores within a trade area has changed. In Section 3, we propose a numerical example to analyze consumer behavior among three trade areas. Furthermore, we forecast consumer behavior among three trade areas when the number of stores has increased. Next we comment on consumer behavior among three trade areas when a large-scale retail store has made inroads into the neighborhood adjacent to the trade areas. In Section 4, we show that the analysis theory can predict consumer behavior among four regions in the Utsunomiya industrial zone.

This study is suggested from the fundamental concept of the Input-Output Table (Leontief, 1986; Miller and Blair, 1985) for consumer behavior between regions, including stores. We feel that the present analysis is very promising for the study of consumer behavior.

2. Input-Output Marketing Analysis Table

2.1. Construction of Fundamental Model

In order to construct the model, the following assumptions must be introduced as elements of the consumer input-output analysis table.

(1) There is a one-to-one corresponding relationship between each consumer and each trade area. That is, one consumer belongs to each trade area.

(2) The number of inflow and outflow consumers from a trade area is proportional to the number of consumers in that trade area.

(3) The sum total of each trade area's number of individual inflow and outflow consumers is equal to the number of consumers occurring in those trade areas at the same time. In other words, there are no interactions between the consumer behavior of each trade area.

When the city's economic consumption sphere is constructed of trade areas $t_1 \ldots t_n$, we consider the inflow and outflow of consumers among these trade areas. If there is a number x_{ij} of consumers shopping from trade area t_i to trade area t_j, from the linearity assumption, the proportion coefficient a_{ij} is obtained:

$$a_{ij} = \frac{x_{ij}}{C_i} \tag{1}$$

where C_i indicates consumers belonging to trade area t_i. The proportion coefficient a_{ij} shows the rate of consumers shopping from trade area t_i to trade area t_j. For this kind of transition probability a_{ij}, an arrangement of all trade areas t_j's transition coefficient for all the trade area t_i is called a transition probability matrix:

$$A \equiv \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}; \text{ transition probability matrix} \tag{2}$$

Each column of this transition matrix A indicates the probability of each trade area's inflow of consumers. The numerical value indicates the degree of ability to attract consumers from trade area t_j to trade area t_i. The balance equation of consumers is expressed as,

$$\begin{cases} x_{11} + x_{12} + \cdots + x_{1n} = C_1 \\ x_{21} + x_{22} + \cdots + x_{2n} = C_2 \\ \vdots \quad \vdots \quad \vdots \\ x_{n1} + x_{n2} + \cdots + x_{nn} = C_n \end{cases} \tag{3}$$

or, when using the coefficient of transition probability,

$$\begin{cases} a_{11} \cdot C_1 + a_{12} \cdot C_1 + \cdots + a_{1n} \cdot C_1 = C_1 \\ a_{21} \cdot C_2 + a_{22} \cdot C_2 + \cdots + a_{2n} \cdot C_2 = C_2 \\ \vdots \quad \vdots \quad \vdots \\ a_{n1} \cdot C_n + a_{n2} \cdot C_n + \cdots + a_{nn} \cdot C_n = C_n \end{cases} \tag{4}$$

is given. The number of customers Y_j shopping in trade area t_j is,

$$\begin{cases} x_{11} + x_{21} + \cdots + x_{n1} = Y_1 \\ x_{12} + x_{22} + \cdots + x_{n2} = Y_2 \\ \vdots \quad \vdots \quad \vdots \\ x_{1n} + x_{2n} + \cdots + x_{nn} = Y_n \end{cases} \tag{5}$$

When this is expressed via the transition probability coefficient, it becomes,

$$\begin{cases} a_{11} \cdot C_1 + a_{21} \cdot C_2 + \cdots + a_{n1} \cdot C_n = Y_1 \\ a_{12} \cdot C_1 + a_{22} \cdot C_2 + \cdots + a_{n2} \cdot C_n = Y_2 \\ \vdots \quad \vdots \quad \vdots \\ a_{1n} \cdot C_1 + a_{2n} \cdot C_2 + \cdots + a_{nn} \cdot C_n = Y_n \end{cases} \tag{6}$$

When this equation (6) is expressed by using the transition matrix A and the consumer matrix C, the following equation (7) is attained:

$$Y \equiv \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix} = {}^tA \cdot C = {}^t\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{pmatrix} = \begin{pmatrix} a_{11} \cdot C_1 + a_{21} \cdot C_2 + \cdots + a_{n1} \cdot C_n \\ a_{12} \cdot C_1 + a_{22} \cdot C_2 + \cdots + a_{n2} \cdot C_n \\ \vdots \quad \vdots \quad \vdots \\ a_{1n} \cdot C_1 + a_{2n} \cdot C_2 + \cdots + a_{nn} \cdot C_n \end{pmatrix} \tag{7}$$

where t indicates the transposed matrix. By using the transition matrix in this way, we can forecast the number of consumers shopping from trade area t_i to trade area t_j. Also, the consumer behavior among trade areas can be calculated by the following equation:

Table 1: Input-Output Marketing Analysis Table

Inflow \ Outflow		Trade areas t_1	t_2	· · ·	t_n	Number of outflow consumers
Trade areas	t_1	x_{11}	x_{12}	· · ·	x_{1n}	C_1
	t_2	x_{21}	x_{22}	· · ·	x_{2n}	C_2
	:	:	:		:	:
	:	:	:		:	:
	t_n	x_{n1}	x_{n2}	· · ·	x_{nn}	C_n
Number of inflow consumers		Y_1	Y_2	· · ·	Y_n	

$$X \equiv \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nn} \end{pmatrix} = C^t \cdot A$$
$$= \begin{pmatrix} C_1 & 0 & \cdots & 0 \\ 0 & C_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & C_n \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \tag{8}$$
$$= \begin{pmatrix} a_{11} \cdot C_1 & a_{12} \cdot C_1 & \cdots & a_{1n} \cdot C_1 \\ a_{21} \cdot C_2 & a_{22} \cdot C_2 & \cdots & a_{2n} \cdot C_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} \cdot C_n & a_{n2} \cdot C_n & \cdots & a_{nn} \cdot C_n \end{pmatrix}$$

Matrix X, which contains the element x_{ij}, represents the input-output marketing analysis table as shown in Table 1. It is possible to forecast consumer behavior among trade areas or the number of customers accumulated within each trade area. Furthermore, when the number of consumers in each trade area changes, we can forecast consumer behavior among trade areas and the number of customers shopping in each trade area.

2.2. Forecast of Consumer Behavior among Trade Areas

When the number of retail stores within a trade area has changed, the ability to attract consumers into its trade area and the number of consumers shopping in its trade area increase. In the case of information related to the transition matrix being available, if the attraction index of a trade area, that is, the ability to attract consumers to a trade area, is tentatively examined, the transition probability matrix can be created using direct and indirect information.

Presently, if the attraction index of a trade area is defined as the sum total of the transition probability of consumers shopping from trade areas t_j to trade area t_i, the attraction index $F(f_j)$ is shown as follows:

$$F(f_j) \equiv \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix} = {}^tA \cdot \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}; \quad \text{when } \sum_{j=1}^{n} f_j = n \tag{9}$$

If the attraction index F is changed to F', it becomes,

$$F'(f_j') \equiv \begin{pmatrix} f_1' \\ f_2' \\ \vdots \\ f_n' \end{pmatrix}; \quad \text{when } \sum_{j=1}^{n} f_j' = n \tag{10}$$

The new transition matrix A' can be found upon using the following recurrent equation with modification diagonal matrices R and S:

$$A_{(h+1)} = A_{(h)} \cdot S_{(h)},\ A_{(h+2)} = R_{(h+1)} \cdot A_{(h+1)},\ (h = 0,1,2,\cdots) \tag{11}$$
$$A_{(0)}, R_{(0)}, S_{(0)}: \ \text{initial value}$$

when

$$A_{(0)} \equiv \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

$$R_{(h)} \equiv \begin{pmatrix} \dfrac{1}{(h)\sum_{j=1}^{n} a_{1j}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \dfrac{1}{(h)\sum_{j=1}^{n} a_{nj}} \end{pmatrix}, S_{(h)} = \begin{pmatrix} \dfrac{f_1'}{(h)\sum_{i=1}^{n} a_{i1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \dfrac{f_n'}{(h)\sum_{i=1}^{n} a_{in}} \end{pmatrix}$$

The most recently calculated transition matrix A' and the consumer matrix C forecast the number of customers shopping in each trade area by using the following equation:

$$Y' \equiv \begin{pmatrix} Y_1' \\ Y_2' \\ \vdots \\ Y_n' \end{pmatrix} = {}^tA' \cdot C = {}^t\begin{pmatrix} a_{11}' & a_{12}' & \cdots & a_{1n}' \\ a_{21}' & a_{22}' & \cdots & a_{2n}' \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}' & a_{n2}' & \cdots & a_{nn}' \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}' \cdot C_1 + a_{21}' \cdot C_2 + \cdots + a_{n1}' \cdot C_n \\ a_{12}' \cdot C_1 + a_{22}' \cdot C_2 + \cdots + a_{n2}' \cdot C_n \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{1n}' \cdot C_1 + a_{2n}' \cdot C_2 + \cdots + a_{nn}' \cdot C_n \end{pmatrix} \tag{12}$$

where t indicates the transposed matrix. The most recently calculated new transition matrix A' finds the input-output marketing analysis table as shown in Table 2, which forecasts consumer behavior by the following equation:

$$X' \equiv \begin{pmatrix} x_{11}' & x_{12}' & \cdots & x_{1n}' \\ x_{21}' & x_{22}' & \cdots & x_{2n}' \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}' & x_{n2}' & \cdots & x_{nn}' \end{pmatrix} = C^{t} \cdot A'$$

$$= \begin{pmatrix} C_1 & 0 & \cdots & 0 \\ 0 & C_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & C_n \end{pmatrix} \cdot \begin{pmatrix} a_{11}' & a_{12}' & \cdots & a_{1n}' \\ a_{21}' & a_{22}' & \cdots & a_{2n}' \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}' & a_{n2}' & \cdots & a_{nn}' \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}' \cdot C_1 & a_{12}' \cdot C_1 & \cdots & a_{1n}' \cdot C_1 \\ a_{21}' \cdot C_2 & a_{22}' \cdot C_2 & \cdots & a_{2n}' \cdot C_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}' \cdot C_n & a_{n2}' \cdot C_n & \cdots & a_{nn}' \cdot C_n \end{pmatrix} \tag{13}$$

Table 2: Input-Output Marketing Analysis Table

Inflow \ Outflow		Trade areas						Number of outflow consumers
		t_1	t_2	·	·	·	t_n	
Trade areas	t_1	x'_{11}	x'_{12}	·	·	·	x'_{1n}	C_1
	t_2	x'_{21}	x'_{22}	·	·	·	x'_{2n}	C_2
	:	:	:				:	:
	:	:	:				:	:
	t_n	x'_{n1}	x'_{n2}	·	·	·	x'_{nn}	C_n
Number of inflow consumers		Y'_1	Y'_2	·	·	·	Y'_n	

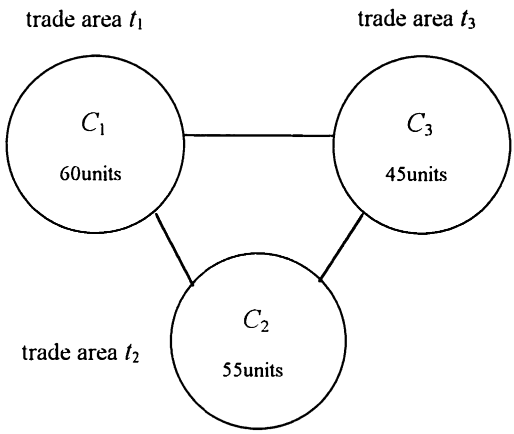

Figure 1: Structure of Three Trade Areas

3. Numerical Example for Three Trade Areas

3.1. Consumer Behavior among Three Trade Areas

We explain how consumer behavior is forecasted by using the numerical example of the simplest case of three trade areas. There are three trade areas, that is, trade areas t_1, t_2, and t_3, as shown in Figure 1. The number of consumers belonging to these trade areas is 60 units, 55 units, and 45 units, respectively. However, one unit can have up to 10,000 people or number of households.

According to the market survey for a definite period of time, the number of

Table 3: Consumer Inflow-Outflow Table in Market Survey

Inflow	Outflow	Trade areas t_1	t_2	t_n	Number of outflow consumers
	t_1	e_{11}	e_{12}	e_{13}	E_1
.		(105)	(45)	(30)	(180)
Trade areas	t_2	e_{21}	e_{22}	e_{23}	E_2
		(45)	(75)	(45)	(165)
	t_3	e_{31}	e_{32}	e_{33}	E_3
		(45)	(30)	(60)	(135)

consumers shopping from trade area t_1 to trade area t_1 was 105. The numbers of consumers shopping from trade area t_1 to trade area t_2 and from trade area t_1 to trade area t_3 were 45 and 30, respectively. At the same time, we can also find the number of consumers shopping from trade area t_2 and trade area t_3, as shown in Table 3. The coefficient of transition probability a_{ij} is each element e_{ij} of the consumer inflow-outflow table shown in Table 3 divided by the number of outflow consumers E_j as follows:

$$A \equiv \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} \frac{e_{11}}{E_1} & \frac{e_{12}}{E_1} & \frac{e_{13}}{E_1} \\ \frac{e_{21}}{E_2} & \frac{e_{22}}{E_2} & \frac{e_{23}}{E_2} \\ \frac{e_{31}}{E_3} & \frac{e_{32}}{E_3} & \frac{e_{33}}{E_3} \end{pmatrix} = \begin{pmatrix} 0.5833 & 0.2500 & 0.1667 \\ 0.2727 & 0.4545 & 0.2727 \\ 0.3333 & 0.2222 & 0.4444 \end{pmatrix} \tag{14}$$

From the transition matrix A and the consumer matrix C, we can find the number of customers shopping in trade areas t_1, t_2 and t_3, as found in the following equation:

$$Y \equiv \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = {}^tA \cdot C = {}^t\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 65 \\ 50 \\ 45 \end{pmatrix}, \tag{15}$$

where t indicates the transposed matrix. For that reason, it is forecasted that the number of customers shopping in trade area t_1 is 65 units, in trade area t_2 is 50 units

Table 4: Consumer Input-Output Marketing Analysis Table

Inflow \ Outflow		Trade areas t_1	t_2	t_3	Number of outflow consumers
Trade areas	t_1	x_{11} (35)	x_{12} (15)	x_{13} (10)	C_1 (60)
	t_2	x_{21} (15)	x_{22} (25)	x_{23} (15)	C_2 (55)
	t_3	x_{31} (15)	x_{32} (10)	x_{33} (20)	C_3 (45)
Number of inflow consumers		Y_1 (65)	Y_2 (50)	Y_3 (45)	

and in trade area t_3 is 45 units. The following equation (16) can be used to forecast these consumer behaviors among the three trade areas.

$$X \equiv \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = C^t \cdot A$$

$$= \begin{pmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 35 & 15 & 10 \\ 15 & 25 & 15 \\ 15 & 10 & 20 \end{pmatrix} \tag{16}$$

Matrix X, which has the elements x_{ij}, arrives to the input-output marketing analysis table as shown in Table 4. In this way the transition matrix, which introduces the ability to attract consumers to a trade area, can be used to calculate these consumer behaviors between trade areas.

3.2. Consumer Behavior with Re-equipment of Shopping Center

When the shopping center within trade area t_3 has re-equipped, or the number of stores within trade area t_3 has changed, we are able to forecast the changes in consumer behavior among the three trade areas. The attraction matrix of ability to attract consumers is based on the definition in equation (9):

$$F(f_j) \equiv \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = {}^tA \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.1894 \\ 0.9268 \\ 0.8838 \end{pmatrix}, \tag{17}$$

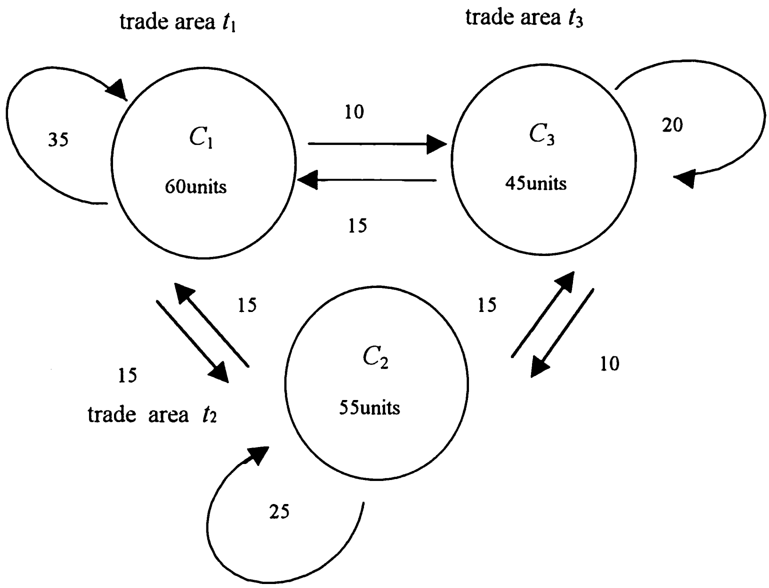

Figure 2: Consumer Input-Output Diagram

where $F(f_j)$ indicates a relative attraction index of each trade area. When the shopping center within trade area t_3 has been re-equipped, and the attraction index of trade area t_3 is changed from 0.8838 to 1.2, a new attraction index $F'(f_j')$ is calculated as follows:

$$F'(f_j') \equiv \begin{pmatrix} f_1' \\ f_2' \\ f_3' \end{pmatrix} = \begin{pmatrix} 1.0117 \\ 0.7883 \\ 1.2 \end{pmatrix}; \quad \text{when} \sum_{j=1}^{3} f_j' = 3 \tag{18}$$

The new transition matrix A' can be sought by using the following recurrent equation:

$$A_{(h+1)} = A_{(h)} \cdot S_{(h)}, \qquad A_{(h+2)} = R_{(h+1)} \cdot A_{(h+1)} \qquad (h = 0,1,2,\cdots) \tag{19}$$

In this way, the transition matrix A' can be obtained by modification matrices R and S.

$$A' \equiv \begin{pmatrix} 0.5243 & 0.2271 & 0.2486 \\ 0.2302 & 0.3878 & 0.3820 \\ 0.2573 & 0.1734 & 0.5694 \end{pmatrix} \tag{20}$$

Table 5: Consumer Input-Output Marketing Analysis Table

Inflow	Outflow	Trade areas t_1	Trade areas t_2	Trade areas t_3	Number of outflow consumers
	t_1	x'_{11}	x'_{12}	x'_{13}	C_1
		(31)	(14)	(15)	(60)
Trade areas	t_2	x'_{21}	x'_{22}	x'_{23}	C_2
		(13)	(21)	(21)	(55)
	t_3	x'_{31}	x'_{32}	x'_{33}	C_3
		(11)	(8)	(26)	(45)
Number of inflow consumers		Y'_1	Y'_2	Y'_3	
		(55)	(43)	(52)	

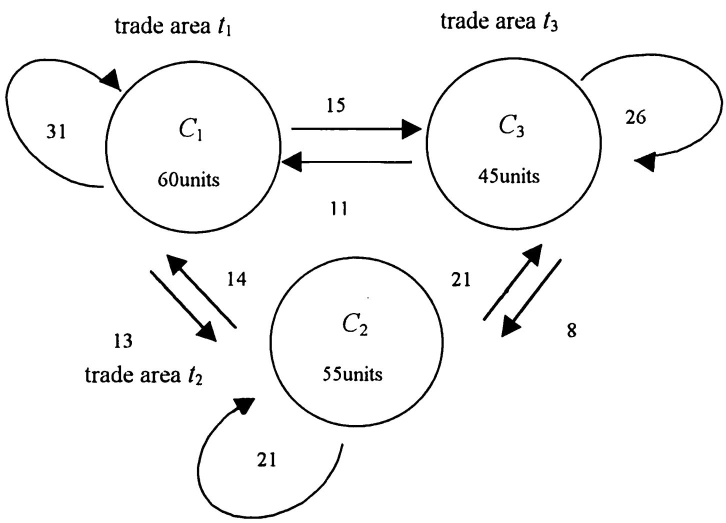

Figure 3: Consumer Input-Output Diagram

In accordance with the most recently calculated transition matrix A', the input-output marketing analysis table, which forecasts consumer behavior, can be found as follows:

$$X' \equiv \begin{pmatrix} x'_{11} & x'_{12} & x'_{13} \\ x'_{21} & x'_{22} & x'_{33} \\ x'_{31} & x'_{32} & x'_{33} \end{pmatrix} = C^{t} \cdot A' = \begin{pmatrix} 31 & 14 & 15 \\ 13 & 21 & 21 \\ 11 & 8 & 26 \end{pmatrix} \tag{21}$$

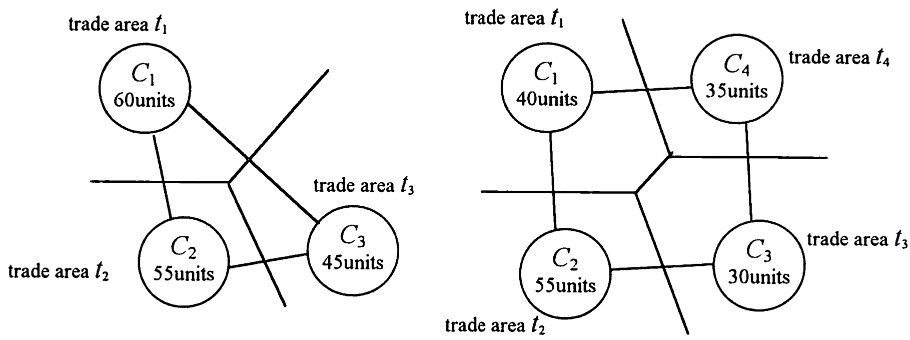

Figure 4: Configuration of Present and Future Trade Areas

Tables 4 and 5 and Figures 2 and 3 show that because the attraction index of trade area t_3 had increased, the consumers in trade area t_3 had not shopped in trade areas t_1 and t_2 but only within trade area t_3. On the other hand, the number of consumers from outside regions shopping in trade area t_3 increased. The decrease of consumers in trade areas t_1 and t_2 is an extremely interesting phenomenon. We can forecast consumer behavior among trade areas by using this technique.

3.3. Influence of Inroads of a Large-scale Retail Store

In this section, we presuppose that a large-scale retail store is scheduled to make inroads into the neighborhood adjacent to trade areas t_1 and t_3. Because the characteristics and primary marketing factors of a large-scale retail store are equivalent to those of a trade area, the large-scale retail store forms a new trade area. Even if the information relating to the inroads of a retail store is scarce, we can obtain the transition matrix which forecasts the changes in consumer behavior among three trade areas and a fourth trade area from existing consumer behavior within the three trade areas.

Suppose that a large-scale retail store has made inroads into the neighborhood adjacent to trade areas t_1 and t_3 and now forms new trade area t_4 as shown in Figure 4. For the sake of this exercise, the number of consumers belonging to trade area t_4 consist of 20 units which have transferred from trade area t_1 and 15 units which have transferred from trade area t_3. There is a great variety in the technique of how consumers move, but they are not discussed in this paper.

In the previous section, the transition matrix A of the present three trade areas has been already found. We estimate the initial value of the transition matrix of the four trade areas in accordance with the foundation of the new trade area t_4. Furthermore, the attraction index of the previous three trade t_1, t_2 and t_3 is as per below. Namely,

$$F(f_j) \equiv \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = {}^tA \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.1894 \\ 0.9268 \\ 0.8838 \end{pmatrix} \tag{22}$$

Here, because the only information related to the trade area t_4 is the relative attraction index, the initial transition probability of trade area t_4 is the average of that of trade areas t_1, t_2 and t_3. However, if the relative information is available, the weighting average is preferred.

$$A_{(0)} \equiv \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dfrac{a_{12}+a_{13}}{2} \\ a_{21} & a_{22} & a_{23} & \dfrac{a_{21}+a_{23}}{2} \\ a_{31} & a_{32} & a_{33} & \dfrac{a_{31}+a_{32}}{2} \\ \dfrac{a_{21}+a_{31}}{2} & \dfrac{a_{12}+a_{32}}{2} & \dfrac{a_{13}+a_{23}}{2} & \dfrac{a_{11}+a_{22}+a_{33}}{3} \end{pmatrix}$$

$$= \begin{pmatrix} 0.5833 & 0.2500 & 0.1667 & 0.2083 \\ 0.2727 & 0.4545 & 0.2727 & 0.2727 \\ 0.3333 & 0.2222 & 0.4444 & 0.2778 \\ 0.3030 & 0.2361 & 0.2197 & 0.4941 \end{pmatrix} \tag{23}$$

Supposing the large-scale retail store makes inroads and trade area t_4 is formed, and the attraction index f_4' of its trade area is 1.2, the attraction index $F'(f_j')$ is calculated as follows:

$$F'(f_j') \equiv \begin{pmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{pmatrix} = \begin{pmatrix} 1.1101 \\ 0.8650 \\ 0.8249 \\ 1.2 \end{pmatrix}; \quad \text{when } \sum_{j=1}^{4} f_j' = 4 \tag{24}$$

The new transition matrix A' is obtained from the initial transition matrix A_0, the new attraction index $F'(f_j')$, and the recurrent equation (11).

$$A' \equiv \begin{pmatrix} 0.4554 & 0.1975 & 0.1331 & 0.2140 \\ 0.1990 & 0.3357 & 0.2035 & 0.2619 \\ 0.2418 & 0.1632 & 0.3298 & 0.2652 \\ 0.2139 & 0.1687 & 0.1586 & 0.4589 \end{pmatrix} \tag{25}$$

Newly calculated transition matrix A' introduces the consumer input-output marketing analysis table and examines the influence of the existing three trade areas in which the large-scale retail store has made inroads.

$$X' \equiv \begin{pmatrix} x_{11}' & x_{12}' & x_{13}' & x_{14}' \\ x_{21}' & x_{22}' & x_{23}' & x_{24}' \\ x_{31}' & x_{32}' & x_{33}' & x_{34}' \\ x_{41}' & x_{42}' & x_{43}' & x_{44}' \end{pmatrix} = C^t \cdot A' = \begin{pmatrix} 18 & 8 & 5 & 9 \\ 11 & 19 & 11 & 14 \\ 7 & 5 & 10 & 8 \\ 7 & 6 & 6 & 16 \end{pmatrix} \tag{26}$$

The consumer input-output marketing analysis table shown in Table 6 can be used to forecast changes of consumer behavior occurring within each trade area. When a large-scale retail store has made inroads into the neighborhood adjacent to trade areas t_1 and t_3, the consumer behavior among each trade area can be forecasted by using this table. Next we will show how diagrams use the consumer behavior before and after inroads into the trade area. Consumer behavior before and after inroads of a large-scale retail store are shown in the following diagram. Although trade area t_2 is positioned relatively far from trade area t_4, many consumers are still shopping in trade area t_4. Similarly, consumers shopping from trade areas t_1 and t_3 to trade area t_2 prior to inroads of the large-scale retail store have decreased significantly, and consumers shopping in trade area t_4 have increased. We cannot conclude that the influence of inroads of the large-scale retail store is weak just because it is far away. This influence

Table 6: Consumer Input-Output Marketing Analysis Table

Inflow \ Outflow		Trade areas t_1	t_2	t_3	t_4	Number of outflow consumers
Trade areas	t_1	x'_{11} (18)	x'_{12} (8)	x'_{13} (5)	x'_{14} (9)	C_1 (40)
	t_2	x'_{21} (11)	x'_{22} (19)	x'_{23} (11)	x'_{24} (14)	C_2 (55)
	t_3	x'_{31} (7)	x'_{32} (5)	x'_{33} (10)	x'_{34} (8)	C_3 (30)
	t_4	x'_{41} (7)	x'_{42} (6)	x'_{43} (6)	x'_{44} (16)	C_4 (35)
Number of inflow consumers		Y'_1 (43)	Y'_2 (38)	Y'_3 (26)	Y'_4 (38)	

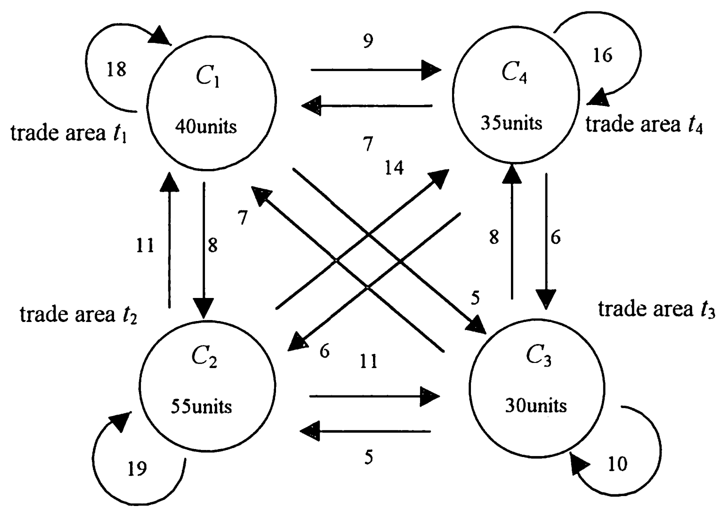

Figure 5: Consumer Inflow-Outflow Diagram

of the large-scale retail store first spreads to the most adjacent trade area and then spreads to the next most adjacent trade area. In this way, we can understand the process by which this influence extends to one trade area after another.

When the large-scale retail store makes inroads into the neighborhood adjacent to trade areas t_1 and t_3, in the same way as other trade areas new trade area t_4 is formed according to retail characteristics and marketing factors of that large-scale retail

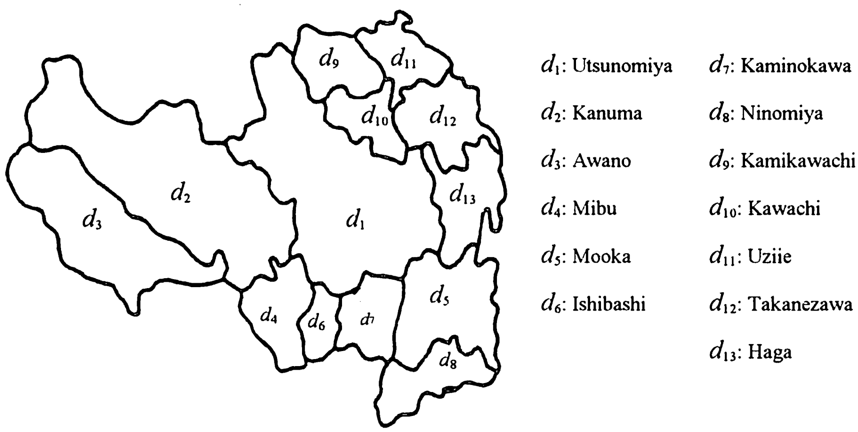

Figure 6: Utsunomiya Industrial Zone

Table 7: Consumer Inflow-Outflow Table in Market Survey

Inflow	Outflow	Utsunomiya industrial zone t_1	t_2	t_3	t_4	Total number of ballots	Number of households
Utsunomiya	t_1	53,066				53,066	5,522
Kanuma	t_2	4,601	13,051	83		17,735	1,944
Mooka	t_3	2,397	151	13,889		16,437	1,760
Kawachi	t_4	7,559		753	6,25	14,570	1,524

store. From results of this analysis we understand that the ability to attract consumers from other trade areas is extremely large. In this case of large-scale retail store inroads, this technique can be extended to quantitatively forecast the influence of changes in consumer behavior within each trade area.

4. Empirical Test for Input-Output Marketing Analysis

When the shopping center has re-equipped, or the number of stores has changed, we demonstrate how consumer behavior within the Utsunomiya industrial zone in 1991 is forecasted from consumer behavior in 1988. The 13 administrative districts in the Utsunomiya industrial zone are shown in Figure 6.

In Table 7 we present the inflow and outflow table based on the market survey for 1988 that has been aggregated into four regions (Tochigi prefectural office, 1988a).

Their regions are Utsunomiya (Utsunomiya), Kanuma (Kanuma, Awano, and Mibu), Mooka (Mooka, Ishibashi, Kaminokawa, and Ninomiya), and Kawachi (Kamikawachi, Kawachi, Uziie, Takanezawa, and Haga).

The most right-hand column is the number of households drawn as a random sample from residents, and the next most right-hand column is the total number of ballots used within this zone. They have 10 ballots and can give their ballots to some regions in which they shop.

The transition probability matrix A is obtained from the row element of the consumer inflow-outflow table shown in Table 7 divided by the number of ballots.

$$A \equiv \begin{pmatrix} 1.0000 & 0 & 0 & 0 \\ 0.2594 & 0.7359 & 0.0047 & 0 \\ 0.1458 & 0.0092 & 0.8450 & 0 \\ 0.5188 & 0 & 0.0517 & 0.4295 \end{pmatrix} \tag{27}$$

Matrix A, which has the elements of transition probability, arrives to the input-output marketing analysis table as shown in Table 8. In this way, the transition matrix can be used to calculate these consumer behaviors within the industrial zone with four regions.

In the same way, market research has been also assembled for 1991 as shown in Table 9 (Tochigi prefectural office, 1991a).

When the shopping center has re-equipped, or the number of stores has changed, we are able to forecast the changes in consumer behavior within the Utsunomiya industrial zone with four regions. The attraction matrix $F(f_j)$ of ability to attract consumers is based on the definition in equation (9):

$$F(f_j) \equiv \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} 1.9240 \\ 0.7451 \\ 0.9014 \\ 0.4295 \end{pmatrix} \tag{28}$$

In Table 10 we present the floor space of retail stores for 1988 and 1991 that has been aggregated to four regions (Tochigi prefectural office, 1988b, 1991b).

Now we introduce an assumption of the gravity model, such as Huff's model, that a region's ability to attract consumers is in proportion to total floor space of retail stores existing in the region. When the number of stores has been increased as shown in Table 10, the new attraction index $F'(f'_j)$ is calculated from the rate of increase as follows:

Table 8: Consumer Input-Output Marketing Analysis Table for 1988

Inflow \ Outflow		Utsunomiya industrial zone t_1	t_2	t_3	t_4	Number of households
Utsunomiya	t_1	130,997				130,997
Kanuma	t_2	8,963	25,428	163		34,554
Mooka	t_3	4,423	279	25,635		30,337
Kawachi	t_4	12,984		1,294	10,749	25,027
Number of consumers		157,347	25,707	27,092	10,749	

Table 9: Consumer Input-Output Marketing Analysis Table for 1991

Inflow \ Outflow		Utsunomiya industrial zone t_1	t_2	t_3	t_4	Number of households
Utsunomiya	t_1	143,570	884	295		144,749
Kanuma	t_2	6,094	31,222	213		37,529
Mooka	t_3	4,618	559	29,426	18	34,621
Kawachi	t_4	12,363	15	1,009	14,940	28,327
Number of consumers		166,645	321,680	30,943	14,958	

Table 10: Floor Space of Retail Stores

		Floor space (m^2) 1988	1991	Rate of increase
Utsunomiya	t_1	434,807	535,927	1.2326
Kanuma	t_2	122,634	143,312	1.1686
Mooka	t_3	109,652	124,928	1.1393
Kawachi	t_4	69,217	73,634	1.0638

$$F'(f_j') \equiv \begin{pmatrix} f_1' \\ f_2' \\ f_3' \\ f_4' \end{pmatrix} = \begin{pmatrix} 2.0072 \\ 0.7369 \\ 0.8692 \\ 0.3867 \end{pmatrix} \tag{29}$$

The new transition matrix A' can be sought from equations (27), (29), and the recurrent equation (11), namely,

Table 11: Consumer Input-Output Marketing Analysis Table

Inflow \ Outflow		Utsunomiya industrial zone t_1	t_2	t_3	t_4	Number of households
Utsunomiya	t_1	144,749				144,749
Kanuma	t_2	10,109	27,269	151		37,529
Mooka	t_3	5,924	356	28,341		34,621
Kawachi	t_4	16,053		1,320	10,954	28,327
Number of consumers		176,835	27,625	29,812	10,954	

$$A' \equiv \begin{pmatrix} 1.0000 & 0 & 0 & 0 \\ 0.2694 & 0.7266 & 0.0040 & 0 \\ 0.1711 & 0.0103 & 0.8186 & 0 \\ 0.5667 & 0 & 0.0466 & 0.3867 \end{pmatrix} \tag{30}$$

In accordance with the most recently calculated transition matrix A', the input-output marketing analysis table, which forecasts consumer behavior, can be found in Table 11.

The results shown in Tables 9 and 11 are sufficient to show the excellent agreement of the empirical data with the numerical data calculated by the present model. Thus, present consumer behavior as well as the floor space of retail stores and the distance between regions are of considerable importance in determining future consumer behavior between regions.

5. Conclusions

We have proposed the probability analysis model, the input-output marketing analysis table, as a means of analyzing consumer behavior. We have also considered in theory the fundamental way of thinking behind the closed-system analysis and the three examples of the region with three trade areas, and have applied this analytical model to predict consumer behavior in the Utsunomiya industrial zone.

Previous analyses have merely forecasted consumer behavior among trade areas considering only physical distance. The present analysis calculates the transition probability matrix based on results of market research, and formulates the input-output marketing analysis table from that matrix. Defining the relative attraction index of a trade area, and depending on the relationship among trade areas, we were able to forecast the consumer input and output behavior between each trade area.

Finally, the present analysis theory based upon the input-output marketing analysis table can be widely applied to predicting other economic transport phenomena, such as the transport of goods and the transmission of information among regions. The theoretical analysis based on the open system and the numerical example for consumer behavior among a city, including three trade areas and its competing two cities, shall be presented in a next paper.

References

[1] Converse, Paul D. (1949), "New Laws of Retail Gravitation", *Journal of Marketing*, Vol. 14, pp. 379-84, October.

[2] Huff, David L. (1962), *Determination of Intra-Urban Retail Trade Areas*, Publication of Real Estate Research Program, Graduate School of Business Administration, Division of Research.

[3] Kotlar, Philip (1971), *Marketing Decision Making: A Model Building Approach*, New York: Holt, Rinehart and Winston.

[4] Leontief, Wassily (1986), *Input-Output Economics*, Oxford University Press, New York.

[5] Miller, R. E. and Blair, P. D. (1985), *Input-Output Analysis: Foundations and Extensions*, New Jersey, Prentice-Hall, Inc., Englewood Cliffs.

[6] Reily, William J. (1929), "Method for the Study of Retail Relationships", *University of Texas Bulletin*, November, No. 2944.

[7] White, L. A. and Ellis, J. B. (1971), "A System Construct for Evaluating Retail Market Locations", *Journal of Marketing Research*, Vol. 8, pp. 43-6, February.

[8] Tochigi Prefectural Office (1988a, 1991a), *Research Report of Consumer Behavior in Tochigi Prefecture* (in Japanese).

[9] Tochigi Prefectural Office (1988b, 1991b), *Commerce in Tochigi Prefecture* (in Japanese).