Technical Note

Input-Output Characteristic Analysis

By
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Abstract

Characteristic of the input-output direct consumption coefficient matrix is a powerful tool for economic structure analysis, as well as a bridge to coordinate the technico-economic structure and the social product structure. The social product structure, and the final product structure, can be observed by looking at the characteristic vectors of this matrix; and, therefore, the coordination relationship between one economic sector with others can be discussed quantitatively.

Adjusting product structures or promoting technical progress can improve the effect of social economic activities. By means of characteristic vectors we can point out the limitation of this effect.

Using China's 1981 and 1987 input-output tables and characteristic method, we can provide a structure analysis of 1981 and 1987 China's economy.

1. Basic Concepts

1.1. Economic Structure Vector

A vector, components of which are all non-negative numbers and the sum of all components is equal to 1, is called economic structure vector. In another word, a vector $X = (X_1, \ldots\ldots, X_n)$ is a economic structure vector if $X_i \geq 0$ is real and

$$\sum_{i=1}^{n} X_i = 1$$

In the two dimensional case (when n = 2) all economic structure vectors form an economic structure of a right triangle AOB with two unit length on the right sides AO and BO; and every straight line connecting the vortex point O and any point C located on the hypotenuse AB corresponds to an economic structure vector (ca cb). All economic structure vectors fill this economic structure triangle, as shown in Figure 1.

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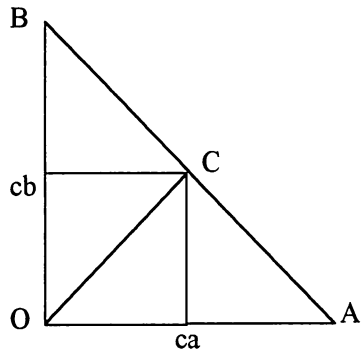


Figure 1: Economic Structure Triangle

Let $F=(F_1,F_2)$ be a vector, and let

$$Fi' = \frac{Fi}{F_1 + F_2} \quad ,$$

then we call the vector $F'=(F_1',F_2')$ as a structurized vector corresponding to the vector $F(Fi \geq 0)$.

1.2. Input-Output Relationship of Structure Type

We will discuss a national economy of n sectors. Let A be a matrix of direct consumption coefficients. We can write the input-output relationship of price type for this economy:

$$\begin{aligned} &AX + Y = X, \\ &A = (a_{ij}), i, j = 1,\ldots\ldots,n \end{aligned} \tag{1}$$

where $Y = (Y_1,\ldots\ldots,Y_n)'$ is a vector of final product, $X = (X_1,\ldots\ldots,X_n)'$ is a vector of social product. Further let $y = (y_1,\ldots\ldots,y_n)'$ and $x = (x_1,\ldots\ldots,x_n)'$ be structurized vectors corresponding to Y and X respectively. We call A the technico-economic structure, while y and x are structures of final product and structure of social product respectively. In this case we can write

$$Y = fy$$
$$X = gx$$

where f is the total value of final product and g is the total value of social product. We can transform the input-output relationship of price type (1) into the input-output relationship of structure type

$$\begin{aligned} &gAx + fy = gx \\ &or \\ &Ax + ry = x \end{aligned} \tag{2}$$

where $r = \frac{f}{g}$ is the ratio of the final product value to the social product value.

1.3. Technico-Economic Structure

The social product structure and the final product structures are all one dimensional structures, but the technico-economic structure is a two dimensional structure, which is a basic structure of social economic activities. The input-output relationship of structure type describes the quantitative relation between the technico-economic structure, the structure of final product and the structure of social product.

From formula (2) we can see that in certain technico-economic structures the final product structure (multiplied by its ratio) can be uniquely determined by the social product structure and inverse the social product structure —by the final product structure (with its ratio); therefore the ratio of final product value —one of the most important indicators of the effect of social economic activities— depends only on the above three mentioned structures.

2. Methodology

2.1. Characteristic Matrix, Characteristic Equation, Characteristic Roots and Characteristic Vectors

Let A be input-output matrix, I —the unit matrix and λ — a parameter. Then

$$\lambda I - A$$

is called the characteristic matrix, and

$$|\lambda I - A| = 0 \tag{3}$$

is the characteristic equation; the solution of which $\lambda = (\lambda_1, \ldots\ldots, \lambda_n)$ are characteristic roots, and for every one of which there is a corresponding characteristic vector R_i of A, that satisfies

$$AR_i = \lambda_i R_i \quad (i = 1, \ldots\ldots, n) \tag{4}$$

It is easy to prove that when $|\lambda_i| < 1$,

$$X = (I - A)^{-1} Y = Y + \sum_{n=0}^{\infty} \lambda_i^n AY = Y + (1 + \lambda_i + \lambda_i^2 + \ldots) AY \tag{5}$$

Here we can see the economic meaning of characteristic roots: $X = Y$ is the first approximation of (5), i.e. the total product is the final one; but in order to produce final product Y it needs AY of productive material (the second approximation); at the same time, in order to produce productive material AY, it needs again $A(AY) = A^2 Y = \lambda_i AY$ of productive material (the third approximation) and so on, and so forth. Thus, we view the productive process of social product X as a infinite process, the characteristic root and its powers are the coefficients of "attenuation" in this process.

2.2. Main Characteristic of Technico-Economic Structure

According to the theorem of nonnegative matrix among characteristic roots, there is only one real characteristic root $\lambda_1, 0 < \lambda_1 < 1$, and $|\lambda_i| < \lambda_1 \ (i = 2, \ldots, n)$. λ_1 is called the main characteristic root of A and the structured vector R_1 corresponding to λ_1 is called the main characteristic vector of A or simply the main characteristic.

2.3. Meaning of the Main Characteristic of Technico-Economic Structure

The main characteristic is a powerful tool of economic analysis and a bridge to co-ordinate the two dimensional technico-economic structure, and the one dimensional social product structure. From (2) we can see that when a technico-economic main characteristic is coincident with a social product structure, then the corresponding final product structure is also coincident with this main characteristic, i.e. when x is the main characteristic and

$$Ax = \lambda_1 x$$

then we can write (2) as

$$\begin{aligned} &\lambda_1 x + ry = x \\ &or \\ &ry = (1-\lambda_1)x, \quad r = 1-\lambda_1 \end{aligned} \tag{5}$$

In this case (when the social product or the final product structure coincides with the main characteristic) the ratio of final product value r depends only on the technico-economic structure, and we can see that the social product structure is co-ordinated completely with the technico-economic structure. Obviously, only in this case is the social product structure coincident completely with the final product structure; and in other cases they may not be coincident with each other.

2.4. Auxiliary Characteristic of Technico-Economic Structure in the Two Sectors' Economies

We can prove that for two sectors input-output relationship the main characteristic vector R_1 of which technico-economic structure must be located in the first quadrant, another technico-economic characteristic vector R_2 must be located in the second or the fourth quadrant. This vector R_2 is also called the auxiliary characteristic of two sectors technico-economic structure. Thus, in the case of two sectors, the main characteristic is the unique direction, along which the social product structure and the final product structure can be coincident.

2.5. Coordinate Region

In the case of two sectors, we can observe the social product structure by using technico-economic characteristics. Meanwhile, we can dispose the social product structure x in a coordinate system in which the bases are the main characteristic R_1 and the auxiliary characteristic R_2, thus

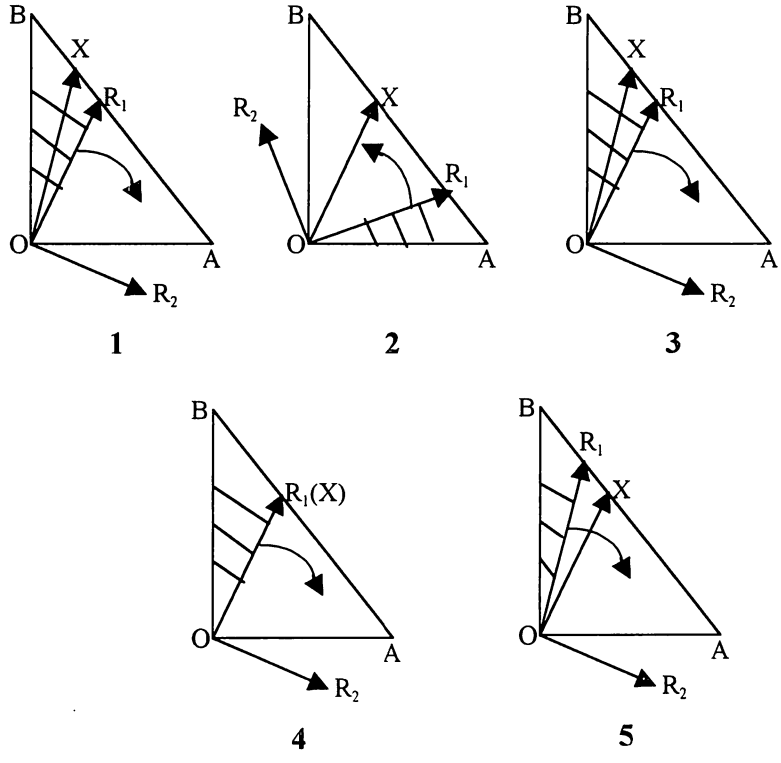
$$x = \alpha_1 \cdot R_1 + \alpha_2 \cdot R_2 \tag{6}$$

It is not difficult to prove that the ratio of final product value is equal to

$$r = \alpha_1 \cdot (1-\lambda_1) + \alpha_2 \cdot (1-\lambda_2) \tag{7}$$

From this formula we can see that the main characteristic R_1 separates the structure triangle AOB into two parts: R_1OA and R_1OB (see Figure 2).

If R_2 is located in the fourth quadrant, when X from R_1 closes to R_2



The above five corresponding figures denote the location of X, R_1 and R_2 in structure triangle for different classification of 1981 China's economy. In these figures, the arrowhead represents the direction of r increase; the region with oblique lines is the non-coordinate region. From these figures, we can see that in 1981, industry and commerce were coordinated with other economic sectors, transportation was basically coordinated with other economic sectors, while agriculture, energy were not well coordinated with other sectors.

In the fourth case of the classification (i.e. the transportation case) the main characteristic vector R_1 is just coincident with the social product structure, in this case the corresponding characteristic root $\lambda_1 = 0.52$ has obvious economic meaning, i.e. that the ratio of final product value is equal to the difference of unity and λ_1, $r = 1 - \lambda_1 = 0.48$.

In Table 2 is shown the effective limitation of adjusting product structure. In 1981 China, reducing the weight of industry in the whole economy or developing agriculture, energy, transportation or commerce would promote the increase of r, meanwhile the effect of rapidly developing agriculture and transportation would be most obvious.

Table 2: Effective Limitation of Structure Adjust

	1	2	3	4	5
max r	.73	.61	.57	.71	.62
min r	.38	.38	.48	.47	.47

Table 3: Characteristics for 1987 Chinese Economy

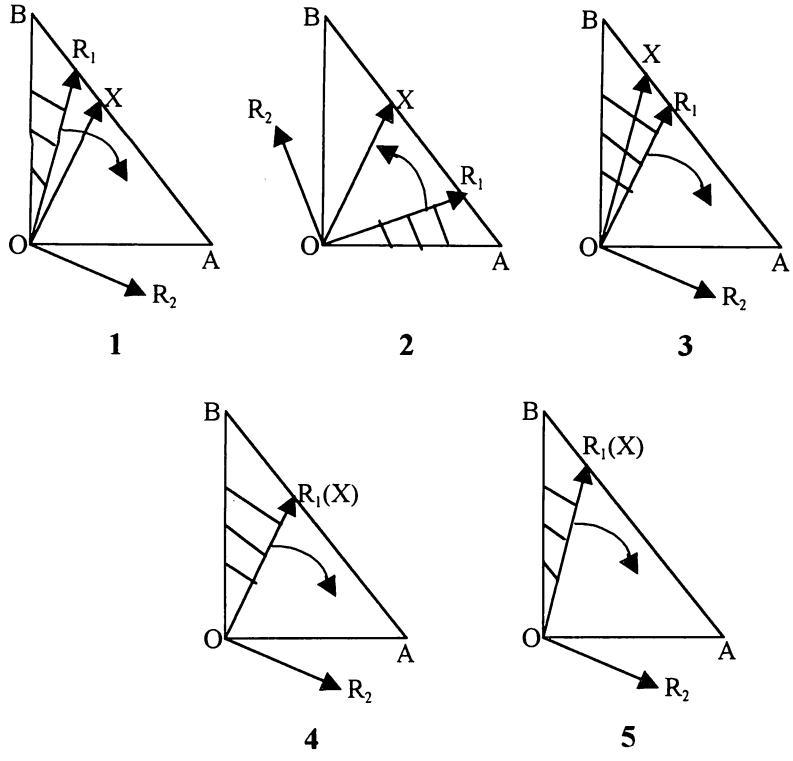
Order	λ_1	λ_2	R_1	R_2	y	x
1	.31	.15	(.33 .67)	(.97 -.03)	(.26 .76)	(.20 .80)
2	.57	.05	(.75 .25)	(-.43 .57)	(.44 .56)	(.60 .40)
3	.54	.25	(.11 .89)	(.41 -.59)	(.02 .98)	(.06 .94)
4	.55	.01	(.03 .97)	(.68 -.32)	(.03 .97)	(.03 .97)
5	.55	.01	(.06 .94)	(.42 -.58)	(.06 .94)	(.06 .94)

4. Structure Analysis of 1987 Chinese Economy

For 1987 Chinese economy the total value of social product was 2291.5 billion RMB (current price). The final product value was 1039.1 billion RMB. The ratio of final product value was r = 0.453. Using the 1987 Chinese input-output table we can calculate all the necessary data for our structure analysis.

For the five mentioned above in B.7 classification (denoted by order numbers 1, 2, 3, 4, 5) the corresponding characteristic roots λ_1 and λ_2 characteristic vectors R_1 and R_2, final product structure y and social product structure x are shown in the above Table 3.

The following five corresponding figures denote the location of X, R_1 and R_2 in the structure triangle for different classification of the Chinese economy in 1987. According to these figures we can conclude that: in 1987 in China, industry and the agriculture were coordinated with other economic sectors; the commerce and transportation sectors were basically coordinated with other economic sectors and the energy sector was not coordinated very well with other sectors.



5. Comparison of 1981 and 1987 Chinese Economic Structure

1. As shown in the above mentioned tables and figures, we can see that characteristic vectors that correspond to five different sector's classification of 1981 and 1987 Chinese economy keep almost the same relative locations. From the viewpoint of characteristic analysis, we can conclude that the structural change of the Chinese economy from 1981 to 1987 is slow and not very obvious.
2. Due to successful agricultural reform, the situation of agriculture in the Chinese national economy had been slightly improved between 1981 and 1987. The agriculture sector changed from a non-coordinate sector in 1981 to a coordinate sector in 1987. The transportation sector also changed its position in the Chinese economic structure between 1981 and 1987.
3. China is a developing country, with the development of the Chinese economy being based mainly on extensive production. At the same time, the macro efficiency of the Chinese economy was even worse from 1981 (then r = 0.48) to 1987 (then r = 0.45). China should focus its efforts especially on

technological progress in order to reduce intermediate consumption.

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