A Dynamic Leontief Pollution Model with Environmental Standards

by

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Abstract

Despite known simplifications the input-output analysis is an effective tool to investigate the environmental impacts of the producing economy on the natural environment. The emission of pollutants is introduced direct in the model studied as joint products of the process of production. The interdependence of the economic activities and the associated pollution emission is analyzed in this paper in a generalized input-output model. The pollution control of the government is defined as an upper bound for the production process, so called environmental standards. The mathematical form of the proposed model consists of a dynamic linear difference equation system of production and a system of inequalities for pollution. We examine the balanced path compared to the environmental standards.

Keywords: Environmental economics, Input-output model, Renewable resources, Pollution emission, Environmental standards

1 Introduction

In this study we will investigate the effects of environmental standards on the trajectories of the activities in an open dynamic Leontief model. Environmental standards are a regulation policy of the government to control the environmental pollution. This method of regulation sets an upper bound on the emission. (Pearce and Turner (1990)) This type of regulation is a good tool to decrease of pollution of renewable resources, like air or water.

The environmental standards in a Leontief model was first examined by Luptáčik and Böhm (1994), but they have investigated a static Leontief model. In the model they have introduced abatement activities, and the net pollution is controlled by environmental standards for the whole economy. Ebiefung and Udo (1999) have shown another environmental application of static Leontief model on industrial pollution emis-

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sion. Wirl (1991) has investigated the effects of environmental standards in a dynamic Arrow-Karlin model. In this model the standards are an upper bound on the production for an one-product firm. In this paper we extend the dynamic Leontief model (Leontief (1986)) with pollution constraints. We analyze the environmental standards in a dynamic context. We assume that the Leontief economy increases in a constant percentage in every time period. It is assumed as well, that the government allows a constantly increasing emission. We will ask, how do these standards influence the balanced growth path of the economy.

The paper organizes, as follows. In section 2 the model is posed and we make some assumption about the model. The next section characterizes the properties of the balanced growth path in case of effective environmental regulation of the government. In section 4 we illustrate our results by the help of a simple numerical example, and the last section summarizes the results of the paper and proposes some possibilities for further research.

2 Description of the model

Our model is based on the equations of the dynamic multi-sector input-output model well known in the literature. (Leontief (1986)) We extend this model with a system of inequalities for pollution emission in order to analyze the balanced path compared to the environmental standards imposed by the government.

Suppose that there are n economic industries each industry producing a single economic commodity and m types of pollutants released by industries. The input-output balance of the economy is described by the equation of economic goods and the inequality of pollutants. The equation of goods describes the balance between the total output of goods of production and the sum of total input of goods of all activities of the economy and the consumed goods.

$$x_t = Ax_t + B(x_{t+1} - x_t) + c_t \tag{1}$$

The inequality of pollution emission describes the control of emission by imposing separate environmental standards on each source of emission. That is, the amount of the emitted pollutants is not allowed to (should not) exceed the prescribed legal limit.

$$Ex_t \leq e_t \tag{2}$$

where

- x_t is the n-dimensional vector of gross industrial outputs,
- c_t is the n-dimensional vector of final consumption demands for economic commodities,
- A is the $n{\times}n$ matrix of conventional input coefficients, showing the input of goods that are required to produce a unit of product,
- B is the $n{\times}n$ matrix of capital coefficients, showing the invested products to

increase the output of the producing sectors by a unit,

- E is the $m \times n$ matrix of pollution output coefficients of producing sectors, showing the quantity of pollutants generated during producing a unit of industrial product,

- e_t is the m-dimensional vector of environmental standards imposed, to limit the pollution emission,

- T is the length of the planning horizon.

Assumption 1.
Throughout the paper it is assumed that the matrices A, B and E are nonnegative, B is nonsingular and c_t is a nonnegative vector. In a previous work Dobos and Floriska (2005) have already studied the balanced growth solution of the system (1) for non-renewable resources corresponding to a given growth rate α ($\alpha \geq 0$) supposing that both production and consumption increase at the same rate α. A similar investigation was made by Schoonbeek (1990). Under these assumptions the balanced growth solution of the model (1) has the form

$$x_t = (1 + \alpha)^t \cdot x_0 \quad \text{and} \quad c_t = (1 + \alpha)^t \cdot c_0 \tag{3}$$

where $\alpha \geq 0$. After substituting the former expressions for x_t and c_t in the equation (1) we have got the following relation

$$(I - A - \alpha \cdot B) \cdot x_0 = c_0 . \tag{4}$$

After that we have established conditions for the existence of nonnegative output configuration x_0. The output configuration x_0 corresponding to equation (4) exists and it is nonnegative if $\alpha \in [0, \alpha_0)$, where α_0 is the marginal growth rate such that $\lambda_1(A + \alpha_0 B) = 1$, i.e. it is the balanced growth rate of the closed dynamic Leontief model. Where $\lambda_1(M)$ denotes the Frobenius root of an arbitrary nonnegative square matrix M, it is the nonnegative real dominant eigenvalue of M. If the former condition for the existence of nonnegative x_0 is fulfilled then the output configuration x_0 has the following form:

$$x_0 (\alpha, c_0) = (I - A - \alpha B)^{-1} \cdot c_0 . \tag{5}$$

3 Properties of the balanced growth path

We assume a balanced growth for the level of the environmental standards as well. Let β denote the rate of growth permitted by government prescribed by the control authority. Then the vector of environmental standards has the form of $e_t = (1 + \beta)^t e_0$ where e_0 denote the initial limit level of emissions. Let us substitute this expression and the relations (3) and (5) in the inequality (2) then we obtain the following inequality

$$\left(\frac{1 + \alpha}{1 + \beta} \right)^t E (I - A - \alpha \beta)^{-1} c_0 \leq e_0 . \tag{6}$$

Lemma 1. The growth rate of production and consumption α is limited by an upper

bound α^* due to environmental regulations. That is the following limitation must hold $0 \leq \alpha \leq \alpha^*$ where

$$\underset{i}{\text{Max}} \left(\frac{E(I - A - \alpha^*B)^{-1}c_0)_i}{(e_0)_i} \right) = 1 \tag{7}$$

and $(\cdot)_i$ denotes the ith component of the respective vector.

Proof. We assume that we are at the beginning of the examined time period i.e. $t = 0$. Using the relation (6), we determine the maximal growth rate α^* for which the quantity of the pollutants generated is not more than the allowed legal limit. Then for this α^* must hold the equality (7).

Remark 1.
By applying environmental standards as the control of pollution, we should impose more strict restriction on the chosen growth rate α than we have made previously (according to Dobos and Floriska (2005) the upper bound for α is the marginal growth rate α_0, i.e. the balanced growth rate for the closed dynamic Leontief model). Considering α_0 for the value of α^* in the equality (7), the left-hand side of it will be an unbounded function for α_0 . This implies that α^* should be less than α_0. That is the following inequalities must hold: $0 \leq \alpha \leq \alpha^* < \alpha_0$.

Assumption 2.
In the next we suppose that at the beginning of the studied time period, the level of pollution emission is less than the legal limit and the growth rate of production α is greater than the growth rate of emission standards β, $\alpha > \beta$. So the environmental standards are defined as

$$e_t = (1 + \beta)^t \cdot e_0 \text{ for } t = 1, 2, ..., T,$$

and e_0 is a given pollution level at the beginning of the planning horizon.

Under these assumptions, there will be exist a time t^* such that the amount of one particular pollutant generated by industries will be equal to the allowed environmental limit.

Lemma 2. The time t^* can be calculated by the following formula:

$$t^* = \frac{1}{\ln \dfrac{1 + \alpha}{1 + \beta}} \underset{i}{\min} \ln \left(\frac{(e_0)_i}{(E)_i(I - A - \alpha B)^{-1}c_0} \right) \tag{8}$$

where $(\cdot)_i$ denotes the ith component of the respective vector and the ith row of the respective matrix.

Proof. By a simple mathematical calculation we express t^* from the inequality (6).

This lemma gives estimation for the time interval without an adjustment process on environmental regulation, i.e. environmental standard. After this point of time the econ-

omy must change either production level or consumption rate, or both. In our model we assume that first the production rate is adjusted to the environmental regulation and then the consumption level. It can be proven that this kind of adjustment process leads to a higher consumption level then another choice, i.e. first adjusted consumption and than production.

Lemma 3. After the time t^* (i.e. for $t \geq t^*$), the *maximum* growth rate of production is β.

Proof. Denote γ the growth rate of production after the time t^*. Then the balanced growth path of production has the form $x_t = (1 + \gamma)^{t-t^*} x_t$. for $t \geq t^*$. The balanced growth path of the environmental standards for the prescribed growth rate β has the form $e_t = (1 + \beta)^{t-t^*} e_t$. for $t \geq t^*$. Substituting these expressions for x_t and e_t the inequality (2) we obtain

$$\left(\frac{1 + \beta}{1 + \gamma} \right)^{t-t^*} e_{t^*} \geq E x_{t^*} \quad \text{for } t \geq t^*.$$

If $\gamma > \beta$, for $t \to \infty$ the former inequality will be $0 \geq E x_{t^*}$. This obviously is not fulfilled for every $x_{t^*} \neq 0$. If $\gamma = \beta$, the former inequality will be $e_{t^*} \geq E x_{t^*}$. This is obviously fulfilled for the time t^*. This concludes that the maximal value for γ, i.e. the maximum growth rate of production is β.

This lemma allows us to construct the path of the production level. The production level is grown with a growth rate α until point of time t^* and after this point the growth rate is β. The growth rate can be determined as follows:

$$x_t = \begin{cases} (1 + \alpha)^t \cdot x_0 & t < t^* \\ (1 + \beta)^{t-t^*} \cdot (1 + \alpha)t^* \cdot x_0 & t^* \leq t \leq T \end{cases} .$$

The next proposition makes it possible to calculate the consumption levels along the planning horizon. Let us now define the growth rate of the production level in the planning horizon as function of the time:

$$\gamma_t = \begin{cases} \alpha & t < t^* \\ \beta & t^* \leq t \leq T \end{cases} .$$

The results of lemmas 1, 2, and 3 can be summarized in the next

Proposition 1. In case of a balanced growth solution of the model (1) and (2), for a given rates of growth, the following must hold

$$(I - A - \gamma_t \cdot B) x_t = c_t, \quad \text{for } t = 1, 2, ..., T . \tag{9}$$

Proof . This relation can be proved in similar way as we have got the relation (4).

The consumption rate can be constructed as

$$c_t = \begin{cases} (1 + \alpha)^t \cdot c_0 & t < t^* \\ (1 + \beta)^t \cdot \left(\dfrac{1 + \alpha}{1 + \beta} \right)^{t^*} (I - A - \beta \cdot B) \cdot (I - A - \alpha \cdot B)^{-1} \cdot c_0 & t^* \leq t \leq T \end{cases}.$$

Remark 2. *An overview of the model.*

The growth rate α of the balanced growth path of the system (1) could be at most α^* according to the Lemma 1. In so far as this rate of growth is greater than the growth rate of environmental standards β, then this balanced path with rate of growth α, can be continued at most to the time t^* according to Lemma 2. After the time t^* the *maximal* growth rate of the balanced path is β, according to Lemma 3. The production corresponding to such a path is growing with a rate of growth α until the time t^*, and with a rate of growth β after this time. In case of different growth rates, to a given level of production correspond different levels of consumption in a given time.

In the next lemma we analyze this change of the consumption level.

Lemma 4. The consumption level at the time t^* is not less than it was at the time t^*-1. That is $c_{t^*} \geq c_{t^*-1}$.

Proof. For $\alpha \geq 0$ the next inequality is obviously fulfilled:

$$c_{t^*} - c_{t^*-1} \geq c_{t^*} - (1 + \alpha)c_{t^*-1} \tag{10}$$

Using the formula (9), Lemma 3 and the Remark 2 we get that

$$c_{t^*} - (1 + \alpha)c_{t^*-1} \geq (I - A - \beta B)x_{t^*} - (1 + \alpha)(I - A - \alpha B)x_{t^*-1} . \tag{11}$$

For the rate of growth α we have

$$(1 + \alpha)x_{t^*-1} = x_{t^*} \tag{12}.$$

By substituting the equation (12) into the inequality (11) finally we obtain the inequality (10) in the following form:

$$c_{t^*}-c_{t^*-1} \geq (\alpha - \beta)Bx_{t^*}$$

The right-hand side of the previous inequality is nonnegative for $\alpha > \beta$, B nonnegative matrix and x_{t^*} nonnegative vector. This concludes that $c_t - c_{t^*-1} \geq 0$.

Remark 3.

The decrease of the growth rate of production from the value α to the value β results an excess supply of economic products in the year t^*. Because the less growth rate of production induces less investments in capital goods. This surplus of goods results a sudden growth of the consumption level in the year t^*.

4 A numerical example

In this section we will demonstrate the functioning of the proposed model. Let us as-

sume that the investigated economy produces three goods and emits two pollutants. The matrices of input coefficients, capital coefficients and emission coefficients are the following:

$$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.5 & 0.3 & 0.1 \\ 0.2 & 0.3 & 0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.07 & 0.03 & 0.02 \\ 0.06 & 0.07 & 0.04 \\ 0.07 & 0.06 & 0.03 \end{bmatrix} \text{ and}$$

$$E = \begin{bmatrix} 0.6 & 0.2 & 0.1 \\ 0.4 & 0.5 & 0.3 \end{bmatrix}.$$

Using the results of assumptions 1 the marginal growth rate of the model is *0.376* (α_o = *0.376*). It means that a rational growth rate must be lower than this growth rate.

Let us assume next that the balanced growth rate α is equal 0.10 i.e. 10% and the vectors of initial consumption level c0 and the initial environmental standards e0 are

$$c_0 = \begin{bmatrix} 5 \\ 7 \\ 6 \end{bmatrix} \text{ and } e_0 = \begin{bmatrix} 90 \\ 130 \end{bmatrix}.$$

We will assume that the growth rate of the environmental standards β is equal to 0.03, i.e. 3 %. The planning horizon of the economy is $T = 35$ years. Applying the equation (5) the initial output of the economy is

$$x_0(0.10) = \begin{bmatrix} 28.215 \\ 35.597 \\ 32.616 \end{bmatrix}.$$

The balanced growth path for the economy is as follows:

$$c_t = \begin{bmatrix} (1+\alpha)^t \cdot c_0 & t < 18 \\ (1+\beta)^{t-19} \cdot c_e & 18 \le t \le 35 \end{bmatrix},$$

where the new initial consumption rate $c_e = (1 + \alpha)^{18} \cdot c_0$ is

$$c_e = \begin{bmatrix} 27.8 \\ 38.919 \\ 33.616 \end{bmatrix}.$$

The pollution for the first type of emission is depicted in Figure 1. with the environmental standards as an upper bound. (Pollution is depicted with a dotted line.) For the second type of pollution the upper bound is not achieved.

The second figure presents production level for the first activity. The dotted line shows balanced growth path in case of no environmental regulation. It can be seen that the

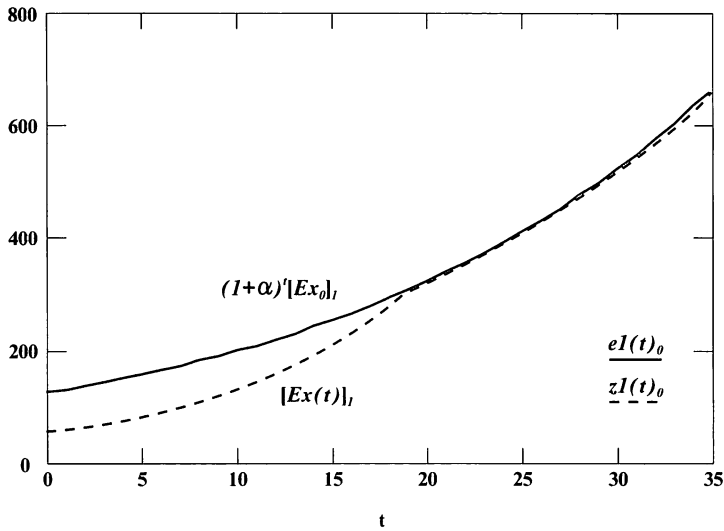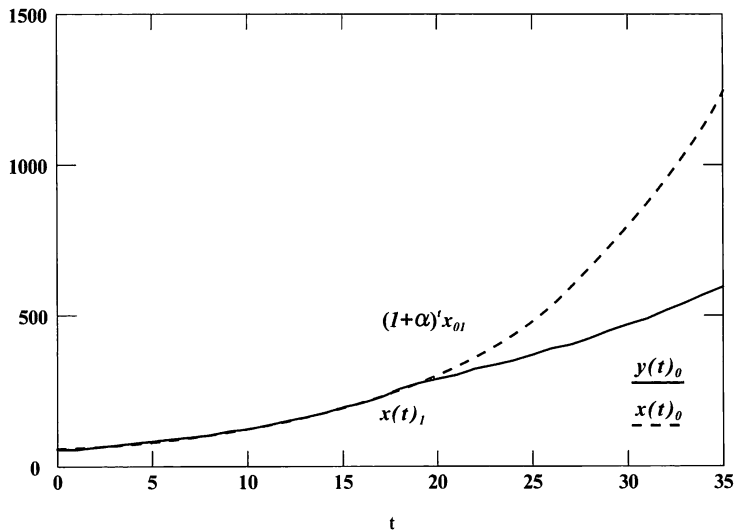**Figure 1: The emission and the environmental standard for pollution 1**

Figure 2: The balaced growth path for the first sector without and with environmental standards

growth path will be lower after the environmental standard is attained.

Figure 3 shows the development of the consumption level in time. The dotted line represents the consumption level in case of environmental standards. This numerical example supports the result of remark 3. After the environmental standards is achieved, the consumption level is higher then without it. But after three time period the consumption level is lower then with environmental standard. The consumption increases because less goods will be invested to increase the production level. It is a positive effect of environmental regulation on the consumption.

Figure 3: The path of the consumption level for the first good without and with environmental standards
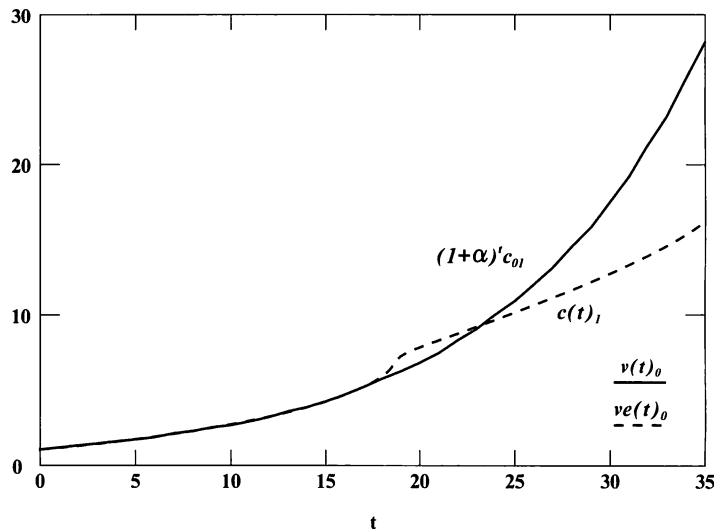
5 Conclusion and further research

In this paper we have investigated the effect of environmental standards on production and consumption in a dynamic Leontief model in case of balanced growth path. If the environmental standards are effective, i.e. it is a constraint on the production, then the growth rate of the production and consumption will be lower after the allowed levels are attained. This means that a rigorous governmental regulation on environmental pollution restricts the emission of harmful materials, if the allowed growth rate of emission is smaller than that of without restriction. In this case the emission standard is a useful tool to avoid such kind of industrial activity of mankind, as global warming. Of course, we can not offer the restriction of consumption in this process. The consumption level is higher after the new growth rate for some period compared to the path without standards.

The investigated model assumes that in the economy there is no technological development. In a further research we could analyze the economy with technological development, i.e. the matrices of the model could be changed in time. In a modern economy the research and development will develop new technologies to save the environment.

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