Does Monetary Policy Work under Zero-Interest-Rate?\textsuperscript{1}

By
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Abstract

In the spring of 1999, the Japanese call money rate reached the zero-interest-rate level, and has remained under quarter percent since then. "Does monetary policy work under zero-interest-rate at all?" is the question to be answered in this study. We are to examine the effectiveness of so-called Quantitative Easing Policy (QEP) introduced by the Bank of Japan in March 2001 using Asset-Liability-Matrix (ALM) derived from the Flow-of-Funds Accounts. The conclusions of this article is that the performance of the QEP conducted by the BOJ is improving gradually in recent days, partially because of the introduction of new measures including corporate stock and ABS purchasing operations.

1 Introduction

Ten years ago, nobody imagined that the interest rate would ever hit the one percent level. Nowadays, we are commonly talking about zero-interest-rate. The U.S. Federal Open Market Committee lowered its intended federal funds rate to one percent on June 25, 2003. Actually the federal funds rate was hovering somewhere around one percent

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for more than one year till the Fed raised the interest rate by 25 basis points to 1.25 percent on June 30, 2004. As early as in the spring of 1999, the Japanese call money rate, an equivalent of U.S. federal funds rate, reached to the zero-interest-rate level, and has remained under quarter percent up to now. (See Figure 1.) In June 2003, even the key long-term interest rate, the yield of the Japanese Government Bond (JGB), hit 0.43 percent for a brief time. Keynes (1936) suggests that the interest rate oriented monetary policy become ineffective at near zero interest rate level because of the existence of the liquidity trap. How about some other type of monetary policy, then? “Does monetary policy work under zero-interest-rate at all?” is the question to be answered in this study.

As Bernanke and Reinhart (2004) suggests, there are three alternative monetary strategies for stimulating the economy that do not involve changing the current value of the policy rate. Specifically, these alternatives involve (a) providing assurance to financial investors that short rates will be lower in the future than they currently expect, (b) shifting the composition of the central bank’s balance sheet, and (c) increasing the size of the central bank’s balance sheet beyond the level needed to set the short-term policy rate at zero. By these definitions, so-called Quantitative Easing Policy (QEP) then newly introduced by the Bank of Japan (BOJ) in March 2001, was a combination of (b) and (c). (See the following section for the details.) If it is the case, the money market operations conducted by the monetary authorities should be fully reflected in their own balance sheets.

In the System of National Accounts (SNA), the financial surplus (i.e. the increment in the difference between the financial assets and the liabilities excluding the changes in the market value) is corresponding to the balance of savings and invest-
ments in the non-financial economy. Thus, if there are induced changes in the assets and/or the liabilities of the economic principals (i.e., institutional sectors including corporations, households, governments etc.) as results of the shifting in composition and/or the changes in the size of the central bank’s balance sheet, the non-financial economy will be affected as well in terms of capital formation and so on. This might be the channel that the changes in the money market operation would affect the non-financial economic activities without changing the current value of the policy rate. Since the Flow-of-Funds Accounts (FFA) is a collection of the balance sheets of the economic principals, by translating those balance sheets into an Asset-Liability-Matrix (ALM) that is a sector-by-sector matrix, we must be able to calculate the induced effects of the QEP on the financial as well as non-financial economy by application of the Leontief inverse commonly used in the input-output analysis. In our experimental study, we were successful to employ the ALM derived from the FFA to examine the impacts of the introduction of the QEP on the stagnated Japanese Economy. (See Tsujimura and Mizoshita (2003).) It was an attempt to apply the concept of the Leontief inverse to the ALM originally proposed by Stone (1966) and Klein (1983).

Since the observation period of the previous study was only a little more than half a year (December 2000 through September 2001), we used the ALM of March 2001 as a benchmark to calculate the effects of the money market operation of the respective month. After the publication of the paper, we have received many useful comments and suggestions to which we are more than grateful. Among them, we found some remarks including the one from Professor Laurence Klein himself to question the stability of the parameters implied in the ALM in a longer period. If the economic structure represented by the ALM is easily changeable, it must be difficult to educate the efficacy of the monetary policy by means of that. The alterations in the non-financial economy could be derived either from the shifts in the money market operation or from the mutation in the flow-of-funds structure of the economy reflected in the coefficient matrix of the ALM. In this new treatise, we are to demonstrate a new procedure to distinguish the former from the latter based on the method of input-output structural decomposition analysis (IO-SDA)\(^2\), so that we can tell the significance of the monetary policy in more precise manner. The expansion of the observation period up to date, which has been made possible without fearing the confusion of the two causes, put us in position to determine if the QEP adopted by the BOJ last four years is a success. This will be a big step forward to examine the usefulness of the monetary policy in a country where zero interest rate prevails.

Klein (2003) hints that the portfolio parameters of an FFA could be a function of the relevant interest rates and the inflation rate. If it is the case, we might be able to construct a model to trace the serial modulation of the ALM itself, which could be a major breakthrough to expand the horizon of the traditional flow-of-funds analysis.

However, when we take only the zero interest rate situations into account, it is a logical contradiction to follow the approach. Actually, in case of today’s Japan, the interest rate remains in the vicinity of nil while the changes in the inflation rate is kept minimal somewhere just below zero. Therefore, we had to develop some other line of procedure to single out the very effects of the QEP.

In the first place, we have abandoned the idea to explain why the economic structure symbolized in the ALM has changed at all. Rather, we opted for a simple and easy scheme. We have just decomposed the observed changes in the non-financial economy into two parts: (1) the first component attributed to the shifts in the money market operation, and (2) the second component attributable to the mutation of the ALM itself. The first component is calculated by multiplying the coefficient matrix of the ALM of the previous period and the money market operation vector of the period; and then subtracting the previous period’s observed value afterwards. We can calculate the second component likewise, by multiplying the coefficient matrix of the ALM of the period and the money market operation vector of the previous period; and then subtracting the previous period’s observed value. The procedure is an analogy to the way we make the NIPA (National Income and Product Accounts) chain index. Moreover, when we take the rate of change into consideration, the geographic mean of the indices of the first component and the second component is consistent with the observed value as Fisher (1927) demonstrated more than seven decades ago.

The second question we are to answer in this study is how effective is each device adopted by the BOJ in its money market operations. In this paper, we are to present the subdivided NII for each market operation instrument. As we discuss in the following sections, the QEP has failed to give favourable results in the early stage of its introduction. Therefore BOJ revised its way of money market operation in the course of trial and error. In the more recent months, the BOJ has introduced drastic measures in face of the prolonged recession and the plunge in the equity prices. One of the most dramatic decisions is to purchase the corporate stocks from the commercial banks, which are obliged to keep the corporate stocks in possession under the value of their equity capital, to cope with the new legislation. Another unprecedented scheme for a central bank is to purchase the Asset-Backed-Securities (ABS) to smooth the financing of the small and medium-sized enterprises to cover up the shortage of the bills in circulation eligible for the BOJ operations. The advantage of the present approach is the capability to single out the effect of a particular policy device upon a particular sector so that we can choose the best combination of the operation instruments.

2 The Quantitative Easing Policy

Before going any further, we have to discuss the details of the QEP adopted by the BOJ. In the spring of 2001, the bank abruptly announced that it would shift the target of the money market operations from the interbank interest rate (overnight call money rate) to the balance of current accounts held by the financial institutions at the central bank. This means that the BOJ expect the commercial banks and other financial institutions to voluntarily build up a balance well over the legally required minimum reserve
in the current account. Simultaneously, the BOJ proclaimed it was to increase the balance of the current accounts (then 4 trillion yen\(^3\)) by one trillion yen to 5 trillion (while keeping the official reserve ratio at the previous level!)\(^4\), and was to add the same amount of JGB on its asset portfolio. The intended amount of the balance of the current accounts was raised to 6 trillion yen in August, then to "above 6 trillion yen" in September, and even further to 10-15 trillion in December 2001. That was not the end of the story. In February 2002, the BOJ announced that it would "provide more liquid-

\(^3\) The balance of the current accounts temporarily increased to 23 trillion yen in December 1999 by way of precaution against so-called Y2K.

\(^4\) At ordinary times, the financial institutions try to keep the balance of current accounts at the level of legally required minimum reserve. Since the BOJ does not pay interest on the current account balances, the banks do not want to pile up "excess-reserve" while paying interest on the deposit accounts with themselves. Of course zero interbank interest rate does not necessarily mean that all the interest rates on the bank accounts become zero. Actually in case of Japan, the banks are paying small amount of interest on the deposits with them while they receive some interest from their borrowers. In that sense it is magic, even under zero-interest-rate circumstances, if BOJ could persuade its customer banks to accumulate as much funds as it wishes. The results are depicted in Fig.2. After the introduction of the QEP in the spring of 2001, the BOJ successfully persuaded the commercial banks to increase the balance of the current accounts not only well above the legally required minimum reserves, but also comfortably above the intended level that they had then proclaimed. One reason must be that Japan is experiencing worst credit crunch ever in the aftermath of the financial bubble of the 1980's, so that the financial institutions are obliged to have excess reserve as a precaution. Another reason could be that the call loan rate (typical interbank interest rate) was in the sub-zero domain from time to time because some foreign banks were able to get profits by borrowing yen against other currencies of higher interest rate and let it to other banks. However these reasoning may explain only a part of the story, and the remainder is left to be answered.
ity to meet a surge in demand irrespective of the target of current account balances, (then) around 10 to 15 trillion yen". The target level was lifted to 15–20 trillion yen in October 2002, 17–22 trillion in March 2003, 22–27 trillion\(^5\) in April, 27–30 trillion in May 2003 and finally to 30–35 trillion yen in January 2004. (See Figure 2.)

Under the zero interest rate situations, the means of the money market operation could have decisive significance. As we have mentioned earlier, in the first phase of the QEP, it was the BOJ's intention to increase JGB in their asset portfolio. This line of policy was officially maintained at least till October 2002, when it announced that it
## Table 1: Balance sheets of the BOJ (100 million yen)

<table>
<thead>
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<tr>
<td><strong>Assets</strong></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Banknotes in circulation</td>
<td>587154</td>
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<td>573999</td>
<td>526885</td>
<td>509865</td>
<td>533928</td>
<td>547499</td>
<td>595647</td>
<td>554347</td>
<td>695920</td>
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<tr>
<td>Current accounts held by financial institutions</td>
<td>34992</td>
<td>58105</td>
<td>40456</td>
<td>57233</td>
<td>43743</td>
<td>61676</td>
<td>46632</td>
<td>60577</td>
<td>238360</td>
<td>138413</td>
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<tr>
<td>Current accounts held by the government</td>
<td>0</td>
<td>4995</td>
<td>49257</td>
<td>5682</td>
<td>6533</td>
<td>5169</td>
<td>20243</td>
<td>24709</td>
<td>35325</td>
<td>59547</td>
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<td>0</td>
<td>0</td>
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<td>Loans to private financial institutions</td>
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<td>70185</td>
<td>51180</td>
<td>49966</td>
<td>90327</td>
<td>79670</td>
<td>51180</td>
<td>37454</td>
<td>36972</td>
<td>15887</td>
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<tr>
<td>Bills purchased and sold</td>
<td>49312</td>
<td>51545</td>
<td>32232</td>
<td>202997</td>
<td>16976</td>
<td>130617</td>
<td>46783</td>
<td>128162</td>
<td>60787</td>
<td>195856</td>
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<td>Repurchase agreement and securities lending transactions</td>
<td>122085</td>
<td>112300</td>
<td>129604</td>
<td>113770</td>
<td>42780</td>
<td>112000</td>
<td>100099</td>
<td>109610</td>
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<td>Financing Bills</td>
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<td>260623</td>
<td>202860</td>
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<td>76809</td>
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<tr>
<td>Central government securities and FILP bonds</td>
<td>327619</td>
<td>346926</td>
<td>362416</td>
<td>387902</td>
<td>399781</td>
<td>401700</td>
<td>451013</td>
<td>443112</td>
<td>435514</td>
<td>453451</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Deposits with agencies</td>
<td>2</td>
<td>202</td>
<td>1</td>
<td>2</td>
<td>202</td>
<td>1</td>
<td>202</td>
<td>1</td>
<td>202</td>
<td>1</td>
</tr>
<tr>
<td>Deposits with depots</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>Corporate stocks</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>839802</td>
<td>880622</td>
<td>1006702</td>
<td>884621</td>
<td>886241</td>
<td>905095</td>
<td>905095</td>
<td>100502</td>
<td>100502</td>
<td>33135</td>
</tr>
</tbody>
</table>

### Notes

- **Table 1:** Balance sheets of the BOJ (100 million yen)
- **Dec-97:** December 1997
- **Mar-98:** March 1998
- **Jun-98:** June 1998
- **Sep-98:** September 1998
- **Dec-98:** December 1998
- **Mar-99:** March 1999
- **Jun-99:** June 1999
- **Sep-99:** September 1999
- **Dec-99:** December 1999
- **Mar-00:** March 2000
- **Assets:** Financial items owned by the BOJ
- **Liabilities:** Financial items owned by others

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Source: Bank of Japan
would increase the monthly outright purchase of the JGB from 1 trillion yen to 1.2 trillion yen. Some other measures included the easing of the restrictions on the use of the Lombard-type lending facility (August 2001 and February 2002), more active purchase of Commercial Papers (December 2001) and the extension of maturities for the bills purchased eligible for the operations (October 2002). More dramatic measures were on their way. In October 2002, the BOJ asked permission to purchase corporate stocks in the form of “money in trust” and the Ministry of Finance (MOF) authorized it immediately. In June 2003, the BOJ announced the scheme for the outright purchases of the ABS, and it was put in practice by the end of the following month.

Figure 3 and Figure 4 as well as Table 1 show the changes in the asset and liability portfolios of the BOJ under the QEP. As indicated in the height of the pillars, the total of the assets and the liabilities have increased gradually since the introduction of the policy in the spring of 2001. It is obvious that the balance of the current accounts has risen dramatically. However, that is not the only cause to make the monetary base grow. The balance of the banknotes has swelled as well, most probably because of the policy shift in April 2002 to allow the liquidation of insolvent financial institutions. On the asset side, there is no doubt that the JGB enlarged its magnitude significantly not only in size, but also in the proportion to the total assets. Another instrument that has expanded its position is bills-purchased in open market operations, especially in more recent days. In contrast to that, the balance of repurchase-agreement and securities-borrowing-transactions has been slashed after the introduction of the QEP.

3 Data

The BOJ publishes Flow of Funds Accounts of Japan quarterly. It contains three tables: (1) financial transactions, (2) financial assets and liabilities, and (3) reconciliation between flows and stocks. The ALM used in this paper has been compiled from the financial assets and liabilities tables of the FFA from December 1997 through December 2004 every three months. Only the summary of the compilation procedure from the FFA to the ALM is shown here, so refer to Tsujimura and Mizoshita (2003) for details. We start from two tables E and R, which are constructed by picking out the assets and liabilities vectors separately from the balance sheets of the FFA. Figure 5 presents the components of the E and R tables.

E is a matrix that shows the portfolio of the fund-employment of each institutional sector, \(e\) and \(sE\) are vectors that represent the excess liabilities and the sum of each row, respectively. \(s\) is the vector that consists of either the sum of the assets or the liabilities, whichever is larger.

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5 The intended amount of the balance of the current accounts was increased to 27~32 trillion yen in October 2003.
6 In our scheme, assets are evaluated in current value while the liabilities are evaluated in book value. For further details, see Tsujimura and Mizoshita (2004).
Figure 5 Components of E− and R− tables

<table>
<thead>
<tr>
<th></th>
<th>institutional sectors</th>
<th>institutional sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>financial institutions</td>
<td>E</td>
<td>$s^E$</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon'$</td>
<td>$s'$</td>
</tr>
<tr>
<td>financial institutions</td>
<td>$R$</td>
<td>$s^R$</td>
</tr>
<tr>
<td></td>
<td>$\rho'$</td>
<td>$s'$</td>
</tr>
</tbody>
</table>

\[
E = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1m} \\
\varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{nm}
\end{bmatrix}
\]

\[\varepsilon = \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_m
\end{bmatrix}
\]

\[s^E = \begin{bmatrix}
s_1^E \\
s_2^E \\
\vdots \\
s_m^E
\end{bmatrix}
\]

\[s = \begin{bmatrix}
s_1 \\
s_2 \\
\vdots \\
s_m
\end{bmatrix}
\]

where, \(n\) denotes the number of financial instruments and \(m\) denotes the number of institutional sectors. \(R\) is a matrix showing the portfolio of the fund-raising of each institutional sector, and \(\rho\) and \(s^R\) are vectors that represent the excess assets and the sum of each row, respectively.

\[
R = \begin{bmatrix}
r_{11} & r_{12} & \cdots & r_{1m} \\
r_{21} & r_{22} & \cdots & r_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
r_{n1} & r_{n2} & \cdots & r_{nm}
\end{bmatrix}
\]

\[\rho = \begin{bmatrix}
\rho_1 \\
\rho_2 \\
\vdots \\
\rho_m
\end{bmatrix}
\]

\[s^R = \begin{bmatrix}
s_1^R \\
s_2^R \\
\vdots \\
s_m^R
\end{bmatrix}
\]

\[s = \begin{bmatrix}
s_1 \\
s_2 \\
\vdots \\
s_m
\end{bmatrix}
\]

It is possible to make out two different sheets of square matrices, the ALM, using \(E\) and \(R\) tables in alternative procedures. One is \(Y\) table based on the fund-raising portfolio, the other is \(Y^*\) table based on the fund-employment portfolio. Superscript * denotes the case of fund-employment. To compile \(Y\)-table in accordance with the fund-raising portfolio, first, matrix \(R\) is substituted for matrix \(U\) (use table or use matrix) and the transposed matrix \(E'\) is substituted for \(V\) (supply table or make matrix).

\[U \equiv R \quad (1)
\]

\[V \equiv E' \quad (2)
\]

In the case of \(Y^*\)-table that represents the fund-employment portfolio, we take matrix \(E\) as \(U'\) and \(R'\) as \(V'\).

\[U' \equiv E \quad (3)
\]

\[V' \equiv R' \quad (4)
\]
Each element of the coefficient matrices $B$ and $B'$ are defined as follows:

\[ b_{ij} = \frac{u_{ij}}{s_j} \]  
\[ b_{ij}' = \frac{u_{ij}'}{s_j} \]  

In the same manner, each element of the coefficient matrices $D$ and $D'$ are defined as follows:

\[ d_{ij} = \frac{v_{ij}}{s_j^e} \]  
\[ d_{ij}' = \frac{v_{ij}'}{s_j^e} \]  

The $m \times m$ coefficient matrices $C$ and $C'$ are estimated using the institutional sector portfolio assumption.

\[ C = DB \]  
\[ C' = D'B' \]  

Then each element of the transaction quantity matrices $Y$ and $Y'$ are obtained as follows,

\[ y_{ij} = c_{ij}s_j \]  
\[ y_{ij}' = c_{ij}'s_j \]  

The above procedure leads us to $Y$ and $Y'$ tables depicted in Fig. 6.

\[ \text{Figure 6 Components of Y- and Y'- tables} \]
4 Methodologies

4.1. Evaluation of the Quantitative Easing Policy

It is necessary to treat BOJ, the central bank, as an exogenous institutional sector in order to analyse the effect of the monetary policy by estimating the induced amount of demand and supply of funds through the intersectoral financial transactions represented in the Leontief inverse. The fundamental equations respect to \( Y \) and \( Y^\ast \) tables are expressed as follows:

\[
C_{boj} s + \varepsilon_j = s
\]

\[
C_{boj} s + \rho_j = s
\]

where \( C_{boj} \) and \( C_{boj}^\ast \) are the matrices obtained from the matrices \( C \) and \( C^\ast \) respectively, by removing the row and the column containing the elements concerning the BOJ. \( \varepsilon_j \) is a \((m-1) \times 1\) vector of which each element is the sum of excess liabilities and the liabilities of the BOJ. \( \rho_j \) is a \((m-1) \times 1\) vector which contains the sum of excess assets and the BOJ’s financial assets. Solving each equation for \( s \) yields

\[
s = (I - C_{boj})^{-1} \varepsilon_j
\]

\[
s = (I - C_{boj})^{-1} \rho_j
\]

where \( I \) denotes \((m-1) \times (m-1)\) unit matrix, \((I - C_{boj})^{-1}\) is the \((m-1) \times (m-1)\) Leontief inverse matrix, which gives the demand for funds as induced by each institutional sector, and \((I - C_{boj})^{-1}\) is the \((m-1) \times (m-1)\) Leontief inverse matrix, by which we can calculate the amount of ultimately induced supply of funds. For simplification, let us denote \((I - C_{boj})^{-1}\) as \( \Gamma \) and \((I - C_{boj})^{-1}\) as \( \Gamma^* \). From the viewpoint of the non-financial economy, the induced demand for funds means gross induced savings (GIS), the amount of new savings required, while the induced supply of funds refers to gross induced investment (GII) that enables us to make still more investments.

It is possible to calculate the effect of the QEP carried by the BOJ using Leontief inverse in the same framework described above. \( \varepsilon_{boj} \) is a \( n \times 1 \) vector of which element \( \varepsilon_{boj,i} \) is the liability held by the BOJ in the form of financial instrument \( i \). The \( n \times 1 \) vector \( \rho_{boj} \) is the assets vector where element \( \rho_{boj,i} \) denotes the financial instrument \( i \) held by the BOJ. Vectors \( \varepsilon_{boj} \) and \( \rho_{boj} \) then should be transformed into \((m-1) \times 1\) vectors \( f_e \) and \( f_i \), each of which is classified by institutional sector in order to make it possible to use Leontief inverse. The method of transformation adopted here is as follows:

\[
f_e = D_{boj} \varepsilon_{boj}
\]

\[
f_i = D_{boj}^* \rho_{boj}
\]

where \( D_{boj} \) and \( D_{boj}^* \) are \((m-1) \times n\) matrices obtained by omitting the row of BOJ from \( D \) and \( D^* \) respectively. Given \( \varepsilon_{boj} \) and \( \rho_{boj} \) exogenously, the induced savings and the induced investments are calculated as follows:
\[ \eta_s = (I - C_{BOJ})^{-1} f_e \]  
(19)

\[ \eta_i = (I - C_{BOJ})^{-1} f_r \]  
(20)

where \( \eta_s \) is the \((m-1) \times 1\) vector of induced savings and element \( \eta_{si} \) denotes the induced savings generated in institutional sector \( i \), \( \eta_i \) is the \((m-1) \times 1\) vector of induced investments, where element \( \eta_{ii} \) indicates induced investment generated in institutional sector \( i \). Note that GIS \( H_s = \sum_i \eta_{si} \) is the sum of the elements \( \eta_s \), and GII \( H_i = \sum_i \eta_{ii} \) is the sum of the elements of \( \eta_i \). Subtracting GIS from GII, we obtain NII as follows:

\[ H_s = H_i - H_s \]  
(21)

NII calculated by (21) have significant economic meanings, that is whether current policy stimulate the capital formation in the non-financial economy or not.

### 4.2. Decomposition of the changes in NII

We have calculated the GIS, GII and NII for December 1997 through December 2004 quarterly. It is possible to decompose the cause of increasing or decreasing of these indices into two elements. These are 1) the portion attributed to BOJ's monetary policy, that is \( f_e \) and \( f_r \), and 2) the segment attributable to the structural change of the financial market, i.e., \( \Gamma \) and \( \Gamma^* \). Let \( f_e \), \( f_r \) be the \((m-1) \times 1\) vector of liabilities (Assets) held by BOJ at period \( t \), and \( \Gamma \), \( \Gamma^* \) be the \((m-1) \times (m-1)\) Leontief inverse matrix at period \( t \). The decomposition of the output change over a period can be calculated as the first difference of \( H_{ni} \), which is:

\[ \Delta H_{ni} = H_{ni} - H_{ni-1} \]  
(22)

\[ \Delta H_{ni} = (H_i - H_s) - (H_{i-1} - H_{s-1}) \]  
(23)

In the matrix notation, by using equations (19) and (20), (23) could be transformed to:

\[ \Delta H_{ni} = (i^T \Gamma^* f_{e_r} - i^T \Gamma f_{e_s}) - (i^T \Gamma_{i-1}^* f_{r_{i-1}} - i^T \Gamma_{i-1} f_{e_{i-1}}) \]  
(24)

The IO-SDA method we have adopted is the arithmetic average of the Laspeyres and Paasche decomposition that is proved to be exact and non-arbitrary.\(^7\)

\[ \Delta H_{ni} = \frac{1}{2} \left[ (i^T \Gamma^* f_{r_{i-1}} - i^T \Gamma f_{e_{i-1}}) - (i^T \Gamma_{i-1}^* f_{r_{i-1}} - i^T \Gamma_{i-1} f_{e_{i-1}}) \right] + \frac{1}{2} \left[ (i^T \Gamma^* f_{e_r} - i^T \Gamma f_{e_s}) - (i^T \Gamma_{i-1}^* f_{e_r} - i^T \Gamma_{i-1} f_{e_{i-1}}) \right] \]  
(25)

where \( i \) is the \((m-1) \times 1\) vector, which contains a column of 1s. As proved in the Ap-

---

\(^7\) For further details, see Betts (1989). The deference between the traditional IO-SDA and ours is that all the components in (25) are scalars instead of vectors because they are multiplied by \( i \).
Appendix 1, equation (24) could be transformed to (25), so that we can trace the change in the NII to two sources. While the first term of the right hand of equation (25) represents the effects of the changes in the BOJ’s fund-raising or fund-employment portfolio, the second term represents the effects of the mutation in the Leontief inverse. Denoting $\Delta f_t$ as the first term, and $\Delta \Gamma_t$ as the second term, (25) can be simplified as:

$$\Delta H_{nt} = \Delta f_t + \Delta \Gamma_t,$$

(26)

Furthermore, we have another method of decomposition to compare the ratio of the NII at $t$ to the NII at $t-1$, which is based on the well-known Fisher’s ideal formula of index numbers. The decomposition of the NII’s change over a period can then be calculated as the ratio of $H_{nt}$ to $H_{n,t-1}$,

$$\delta H_{nt} = \frac{H_{nt}}{H_{n,t-1}}$$

(27)

which is:

$$\delta H_{nt} = \frac{(H_n - H_{nt})}{(H_{n,t-1} - H_{n,t-1})}$$

(28)

In matrix notation, using equations (19) and (20), (28) can be transformed to:

$$\delta H_{nt} = \frac{\mathbf{i}^T \mathbf{r}_{t-1} - \mathbf{i}^T \mathbf{r}_t}{\mathbf{i}^T \mathbf{r}_{t-1} - \mathbf{i}^T \mathbf{r}_{t-1}}$$

(29)

Expanding (29) yields (see Appendix 2):

$$\delta H_{nt} = \sqrt{\frac{\mathbf{i}^T \mathbf{r}_t - \mathbf{i}^T \mathbf{r}_{t-1}}{\mathbf{i}^T \mathbf{r}_{t-1} - \mathbf{i}^T \mathbf{r}_{t-1}}} \times \sqrt{\frac{\mathbf{i}^T \mathbf{r}_{t-1} - \mathbf{i}^T \mathbf{r}_{t-1}}{\mathbf{i}^T \mathbf{r}_{t-1} - \mathbf{i}^T \mathbf{r}_{t-1}}}$$

(30)

where $\mathbf{i}$ is the unit vector. The first square root on the right hand side of (30) means the effect of the changes in the BOJ’s fund-employment or fund-raising portfolio, and the second square root does that of the changes in the Leontief inverse. Denoting $\delta f_t$ as the first square root, and $\delta \Gamma_t$ as the second square root, (30) can be further simplified as:

$$\delta H_{nt} = \delta f_t \times \delta \Gamma_t,$$

(31)

This relation is originated in the property of the Fisher index (Fisher (1927))

$^9$. Although $\delta H_{nt}$ is the changing rate from $t-1$ to $t$, it can be used as chain index as well. When we calculate the changing rate from term 0 to term $\tau$, that is given by:

$$\prod_{t=1}^{\tau} \delta H_{nt} = \prod_{t=1}^{\tau} \delta f_t \times \prod_{t=1}^{\tau} \delta \Gamma_t,$$

(32)


$^9$ It should be noted that all the components become scalar by multiplying $\mathbf{i}^T$, so that (30) reduces to the original Fisher’s formula.
5 The Results

There is an asymmetry in the propagation of the supply and the demand of the funds in the financial system. The demand for funds should be eventually financed by the gross induced savings (GIS), while the supply of funds brings gross induced investments (GII) in due course. The QEP requires the central bank to choose two items simultaneously, one in assets and another in liabilities. This action changes GII on the one hand and the GIS on the other. The asymmetry in the propagation process gives net induced investments (NII) as a difference between the GII and the GIS. The sign and the amount of the NII is nothing but the indicator of the effectiveness of the monetary policy made on the non-financial economy. A policy that induces a positive number of NII gives an expansion in the economy, so that it will be welcomed, especially in the course of a recession. In contrast, a money market operation that yields a negative NII, i.e., net induced savings (NIS), weakens the economy so that it should be avoided while the depression prevails10.

The fluctuations in the three indices, GII(H), GIS(Hs) and NII(Hn), between March 1998 and December 2004 are depicted in Figure 7 quarterly. Just for information, same three indicators between March 1980 and March 200411 are presented in Figure 8 annually. The first impression might be that the GIS is a mirror image of the GII. Since the asset and the liabilities are matched up on the balance sheet of the BOJ, these two indices fluctuate hand in hand. However, the NII that is the difference between the GII and the GIS is not stable at all. In Figure 8, the NII remains negative between 1980 and 1986 and then stays in the positive domain from 1987 through 1993. The indicator turned into negative in 1994 and remained so since then. As it is obvious in Figure 7, despite the introduction of the QEP in March 2001, the NII stayed in the negative region throughout the period. In that sense, the policy did not help to bail out Japan from its worst recession in more than five decades. But the magnitude of the NII is not stable by any means. After the inauguration of the policy, the NII experienced a set back for more than one year, and the indicator started to show gradual improvement suggesting that the performance of the money market operation was getting better. Especially in 2004, the improvement seems rather obvious.

The quarterly changes in the NII alongside its decomposition are shown in Figure 9. Although the NII stayed in the negative region throughout the period, the indicator moved favourably in and after December 2001. The pillars are divided into two parts: the dotted portion indicates the alteration attributed to the changes in the portfolio of the central bank, and the segment with oblique lines attributable to the mutation of the coefficients of the ALM. As far as the absolute value is concerned, in 23 out of 28 cases (82 per cent), the contributions of the shift in money market operation reflected in the asset and liability portfolio of the central bank are greater than those of the mutation of the ALM. Same thing is demonstrated in Figure 10 in a different manner. The solid line presents the changes in the NII as a proportion to the previous period. Likewise, the broken and dotted lines display those attributable to the BOJ portfolio and

10 For further discussion in this respect, see Appendix 3.

the mutation in ALM respectively. Do mind that the larger the proportion, the absolute magnitude of the NII increases in the negative domain. This picture more or less clearly tells us that the shifts in the portfolio of the central bank have absolute significance to the performance of the non-financial economy in most periods. Actually, in 24 out of 27 cases (90 per cent), the direction of the effects of the money market operation coincides with that of the NII in total. Comparing to this, in only 11 out of 27 cases (41 per cent), the direction of the effects of the mutation of ALM coincides with
that of combined NII. Therefore, it can be said that the money market operation plays
a dominant role in the determination of the NII, which in its turn influence the rise and
the fall of the non-financial economy.

The next question to be answered is what kind of money market operation brings
favourable effects on the NII. Table 2 presents per-unit GII (in the descending order)
and GIS (in the ascending order) generated by each available device of money market operations. Since NII is the difference between GII and GIS, the greater is preferred to the smaller in GII while the smaller is preferred to the greater in GIS. It is obvious at the first glance that the newly introduced device, the purchase of the ABS, is more efficient than any other instruments to push up the NII. Among the traditional money market operation tools, the bill purchasing operation and the loans to the commercial banks are the most powerful of all, followed by repurchase agreement and securities lending transactions by narrow margin. The purchasing of corporate stocks, also a new comer, follows close behind the bill purchasing operation. Unfortunately, JGB, the leading item in the BOJ asset portfolio, gives only small GII. On the liabilities side, bill-selling operation is the best device to raise funds because it gives least burden in terms of GIS\textsuperscript{12}. The issuance of the bank notes as well as the repurchase agreement and the securities lending transactions give relatively smaller GIS. In contrast to that, the current accounts at BOJ held by the financial institutions and the government are a little more burdensome to the economy.

Table 2: Per unit GII and GIS produced by each money market operation device (Dec 2004)

<table>
<thead>
<tr>
<th>Assets of BOJ</th>
<th>GII</th>
<th>Liabilities of BOJ</th>
<th>GIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset-Backed-Securities</td>
<td>4.4386</td>
<td>Bills sold</td>
<td>0.0266</td>
</tr>
<tr>
<td>Loans to private financial institutions</td>
<td>4.1272</td>
<td>Banknotes in circulation</td>
<td>3.0409</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>3.9787</td>
<td>Resale agreement and securities borrowing transactions</td>
<td>3.1190</td>
</tr>
<tr>
<td>Corporate stocks</td>
<td>3.9291</td>
<td>Current accounts held by financial institutions</td>
<td>3.6301</td>
</tr>
<tr>
<td>Repurchase agreement and securities lending transactions</td>
<td>3.1636</td>
<td>Current accounts held by the government</td>
<td>3.7380</td>
</tr>
<tr>
<td>JGB</td>
<td>1.9217</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financing Bills</td>
<td>1.4608</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The amount of GII and GIS produced by increases of 1 unit in asset or liability items.

One advantage of the ALM analysis is that it gives more detailed figures on the sector-by-sector GII and GIS generated by the money market operations. The summarized results are listed in Table 3. The non-financial private enterprises are the largest beneficiary of all in terms of the relative proportion of GII to GIS in general. Both of the newly introduced BOJ’s devices of money market operation, i.e., corporate stock and ABS purchasing operations, give large per unit GII to this sector. In sharp contrast to this, money market operations do not benefit households well; rather they give more burdens in the form of GIS. Unless the central bank sells bills to finance it, any type of money market operation fails to produce positive NII on the households. The financial institutions are affected a lot by the money market operations in either way. The ABS and bill purchasing operations as well as the central bank loan directed to them give

\textsuperscript{12} The problem is that bill-selling operation causes least GIS because most of the available bills are eventually purchased by the BOJ itself.
Table 3: Monetary operation options and GII GIS for each institutional sector (2004Q4)

Nonfinancial private enterprises

<table>
<thead>
<tr>
<th>Assets</th>
<th>GII</th>
<th>Liabilities</th>
<th>GIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate stocks</td>
<td>1.6966</td>
<td>Bills sold</td>
<td>0.0017</td>
</tr>
<tr>
<td>Asset–Backed–Securities</td>
<td>0.9277</td>
<td>Current accounts held by the government</td>
<td>0.1991</td>
</tr>
<tr>
<td>Loans to private financial institutions</td>
<td>0.8342</td>
<td>Repurchase agreement and securities borrowing transactions</td>
<td>0.2297</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>0.7234</td>
<td>Current accounts held by private financial institutions</td>
<td>0.3673</td>
</tr>
<tr>
<td>Repurchase agreement and securities lending transactions</td>
<td>0.5542</td>
<td>Banknotes in circulation</td>
<td>0.6825</td>
</tr>
<tr>
<td>JGB</td>
<td>0.1683</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financing Bills</td>
<td>0.0992</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Households and Private nonprofit institutions serving households

<table>
<thead>
<tr>
<th>Assets</th>
<th>GII</th>
<th>Liabilities</th>
<th>GIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset–Backed–Securities</td>
<td>0.4219</td>
<td>Bills sold</td>
<td>0.0043</td>
</tr>
<tr>
<td>Loans to private financial institutions</td>
<td>0.4182</td>
<td>Resale agreement and securities borrowing transactions</td>
<td>0.5561</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>0.3359</td>
<td>Current accounts held by the government</td>
<td>0.7401</td>
</tr>
<tr>
<td>Corporate stocks</td>
<td>0.3148</td>
<td>Current accounts held by private financial institutions</td>
<td>0.8657</td>
</tr>
<tr>
<td>Repurchase agreement and securities lending transactions</td>
<td>0.2359</td>
<td>Banknotes in circulation</td>
<td>1.0529</td>
</tr>
<tr>
<td>JGB</td>
<td>0.0775</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financing Bills</td>
<td>0.0365</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Financial institutions

<table>
<thead>
<tr>
<th>Assets</th>
<th>GII</th>
<th>Liabilities</th>
<th>GIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset–Backed–Securities</td>
<td>2.5494</td>
<td>Bills sold</td>
<td>0.0193</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>2.3988</td>
<td>Banknotes in circulation</td>
<td>1.1917</td>
</tr>
<tr>
<td>Loans to private financial institutions</td>
<td>2.3276</td>
<td>Current accounts held by the government</td>
<td>1.5841</td>
</tr>
<tr>
<td>Repurchase agreement and securities lending transactions</td>
<td>1.9701</td>
<td>Resale agreement and securities borrowing transactions</td>
<td>2.0725</td>
</tr>
<tr>
<td>Corporate stocks</td>
<td>1.5219</td>
<td>Current accounts held by private financial institutions</td>
<td>2.2129</td>
</tr>
<tr>
<td>JGB</td>
<td>0.6103</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financing Bills</td>
<td>0.2285</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Nonfinancial public institutions

<table>
<thead>
<tr>
<th>Assets</th>
<th>GII</th>
<th>Liabilities</th>
<th>GIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financing Bills</td>
<td>1.0967</td>
<td>Bills sold</td>
<td>0.0013</td>
</tr>
<tr>
<td>JGB</td>
<td>1.0657</td>
<td>Banknotes in circulation</td>
<td>0.1138</td>
</tr>
<tr>
<td>Loans to private financial institutions</td>
<td>0.5472</td>
<td>Current accounts held by private financial institutions</td>
<td>0.1841</td>
</tr>
<tr>
<td>Asset–Backed–Securities</td>
<td>0.5396</td>
<td>Resale agreement and securities borrowing transactions</td>
<td>0.2608</td>
</tr>
<tr>
<td>Bills purchased</td>
<td>0.5206</td>
<td>Current accounts held by the government</td>
<td>1.2148</td>
</tr>
<tr>
<td>Repurchase agreement and securities lending transactions</td>
<td>0.4034</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate stocks</td>
<td>0.3960</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
relatively large GII to the financial institutions. However, the commercial banks’ own current accounts with BOJ give GIS of 2.2; which will offset most of the GII created by any devices of the central bank operation. Although both per unit GII and GIS are generally small in the non-financial public institutions including central and local governments, they tend to be benefited by FB and JGB operations. JGB purchasing operation, BOJ’s prominent operation device, gives significant GII to non-financial public institutions while giving minimal GII to the private sectors.

6 Conclusions

The above-mentioned observation put us in position to determine if the QEP adopted by the BOJ last four years is a success without fearing the confusion of the two causes, i.e., the effects attributed to the BOJ’s monetary policy itself and those attributable to the structural changes in the financial market. The analysis suggests that the effect of the former is obviously greater than that of the latter. This reconfirms the usefulness of the ALM derived from the FFA in the assessment of the effects of the money market operations\textsuperscript{13}. It is proved that the selection of the operation instrument is vital because there are vast differences between their effects on the non-financial economy. Moreover we can tell that each operation method give conflicting net results on different institutional sectors.

To overcome the persistent recession, it is preferable to adopt money market operation devices that create more NII in the non-financial sectors rather than in public sectors. In this regards, the open market operation of JGB, the device the BOJ has selected at the first stage of the QEP, is the least likely candidate. JGB creates relatively large amount of NII in the public sector, but gives only small amount in the private sectors. Comparing to this, those traditional money market operation devices like bill purchasing or lending facilities induces more favourable effects on the private sectors in terms of NII. Although it is criticized as unusual measures for a central bank, the introduction of new instruments including corporate stocks and ABS to BOJ’s asset portfolio widens the opportunity to create more NII in the private sectors.

Four years has passed since the introduction of the QEP by the BOJ that is fighting against the worst recession in the post-war Japan where zero-interest-rate is a matter of fact. The overall performance of the QEP is improving gradually in recent days, partially because of the introduction of the new measures including corporate stock and ABS purchasing operations. As a conclusion it can be said that some type of monetary policy could work even under zero-interest-rate. BOJ, the pioneer in this field, is getting some experiences through trial and error, but still it is a long way to be truly successful.

Appendix 1

\textsuperscript{13} For further investigation in the stability of the parameters involved in the ALM in a longer perspective, see Tsujimura and Mizoshita (2004).
Equation (25) in the main text is obtained through the following manipulation of (24):

\[
\Delta H_{N_0} = (iT_x^* f_{p,1} - iT_x^* f_{x,j}) - (iT_x^* f_{p,1} - iT_x^* f_{x,j}) 
\]

\[
= \frac{2 (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - 2 (iT_x^* f_{p,j} - iT_x^* f_{x,j})}{2} 
\]

\[
+ \frac{2 (iT_x^* f_{p,x} - iT_x^* f_{x,x}) - 2 (iT_x^* f_{p,x} - iT_x^* f_{x,x})}{2} 
\]

\[
+ \frac{2 (iT_x^* f_{p,j} - iT_x^* f_{x,j}) + (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j})}{2} 
\]

\[
(iii) \quad \{ (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j}) \} + \{ (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j}) \} 
\]

\[
= \frac{2 (iT_x^* f_{p,j} - iT_x^* f_{x,j}) + (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j})}{2} 
\]

\[
(iv) \quad \{ (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j}) \} + \{ (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j}) \} 
\]

\[
= \frac{2 (iT_x^* f_{p,j} - iT_x^* f_{x,j}) + (iT_x^* f_{p,j} - iT_x^* f_{x,j}) - (iT_x^* f_{p,j} - iT_x^* f_{x,j})}{2} 
\]

\[
(25)
\]

(i) The differences in NII caused by the transition of \( t \) from \( t-1 \) to \( t \) while \( \Gamma \) and \( \Gamma' \) are kept at \( t-1 \).

(ii) The differences in NII caused by the transition of \( t \) from \( t-1 \) to \( t \) while \( \Gamma \) and \( \Gamma' \) are kept at \( t \).

(iii) The differences in NII caused by the transitions of \( \Gamma \) and \( \Gamma' \) from \( t-1 \) to \( t \) while \( f \) is kept at \( t-1 \).

(iv) The differences in NII caused by the transitions of \( \Gamma \) and \( \Gamma' \) from \( t-1 \) to \( t \) while \( f \) is kept at \( t \).

Therefore the first term of (25) represents the differences in NII caused by the transition of \( f \) from \( t-1 \) to \( t \), equally arithmetically weighted by \( \Gamma \) and \( \Gamma' \) at \( t-1 \) and \( t \). Likewise, the second term of the equation indicates the differences in NII caused by the transitions of \( \Gamma \) and \( \Gamma' \) from \( t-1 \) to \( t \), equally arithmetically weighted by \( f \) at \( t-1 \) and \( t \).

Appendix 2

Equation (30) in the main text is obtained through the following manipulation of (29):

\[
\delta H_{N_0} = \frac{iT_x^* f_{p,j} - iT_x^* f_{x,j}}{iT_x^* f_{p,j} - iT_x^* f_{x,j}} 
\]

\[
= \sqrt{\frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})^2}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})^2}} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} 
\]

\[
= \sqrt{\frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} 
\]

\[
(i) \quad \text{max} \left\{ \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \right\} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} 
\]

\[
(ii) \quad \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} 
\]

\[
(iii) \quad \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} 
\]

\[
(iv) \quad \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} \times \frac{(iT_x^* f_{p,j} - iT_x^* f_{x,j})}{(iT_x^* f_{p,j} - iT_x^* f_{x,j})} 
\]

\[
(30)
\]
(i) The rate of change in NII caused by the transition of \( t \) from \( t-1 \) to \( t \) while \( \Gamma \) and \( \Gamma' \) are kept at \( t-1 \).

(ii) The rate of change in NII caused by the transition of \( t \) from \( t-1 \) to \( t \) while \( \Gamma \) and \( \Gamma' \) are kept at \( t \).

(iii) The rate of change in NII caused by the transitions of \( \Gamma \) and \( \Gamma' \) from \( t-1 \) to \( t \) while \( t \) is kept at \( t-1 \).

(iv) The rate of change in NII caused by the transitions of \( \Gamma \) and \( \Gamma' \) from \( t-1 \) to \( t \) while \( t \) is kept at \( t \).

Therefore the first term of (30) represents the rate of change in NII caused by the transition of \( t \) from \( t-1 \) to \( t \), equally geometrically weighted by \( \Gamma \) and \( \Gamma' \) at \( t-1 \) and \( t \). Likewise, the second term of the equation indicates the rate of change in NII caused by the transitions of \( \Gamma \) and \( \Gamma' \) from \( t-1 \) to \( t \), equally geometrically weighted by \( t \) at \( t-1 \) and \( t \).

Appendix 3

We apply the techniques of cointegration tests developed by Engle and Granger (1987) to examine the casual relationship between the rate of change in the GDP and the NII. For simplification, the former is noted as RGDP. Table A-1 reports the summary of augmented Dickey-Fuller (ADF) test using the annual data from March 1980 to March 2004\(^{14}\). (a) is the null hypothesis that a single unit root exists in RGDP and ARGDP. Based on the ADF – t statistics, the null hypothesis of a unit root in RGDP is not rejected at 1 percent significance level, while that in ARGDP is rejected. (b) is the null hypothesis that a single unit root exists in NII and ANII.

Table A-1: ADF test statistics for RGDP and NII

<table>
<thead>
<tr>
<th>(a) ADF test statistics for RGDP</th>
<th>with time trend</th>
<th>without time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGDP</td>
<td>-2.891439 (0.1822)</td>
<td>-1.679476 (0.4284)</td>
</tr>
<tr>
<td>Lags</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ARGDP</td>
<td>-4.975085** (0.0030)</td>
<td>-5.069016** (0.0005)</td>
</tr>
<tr>
<td>Lags</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) ADF test statistics for NII</th>
<th>with time trend</th>
<th>without time trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>NII</td>
<td>-2.250718 (0.4419)</td>
<td>-0.834724 (0.7908)</td>
</tr>
<tr>
<td>Lags</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>ANII</td>
<td>-4.850579** (0.0043)</td>
<td>-4.477876** (0.0021)</td>
</tr>
<tr>
<td>Lags</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Values in parentheses are MacKinnon (1996) one-sided p-values.

** indicate that the null hypothesis that a single unit root exists can be rejected at 5% significance level. Optimal lag length was chosen based on the Schwarz Information criterion (SIC).

Table A–2: Unrestricted Cointegration Rank Test (Trace)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.468696</td>
<td>15.76984</td>
<td>15.49471</td>
<td>0.0455</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.051833</td>
<td>1.224157</td>
<td>3.841466</td>
<td>0.2685</td>
</tr>
</tbody>
</table>

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level

Table A–3: Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Max–Eigen Statistic</th>
<th>0.05 Critical Value</th>
<th>Prob.**</th>
</tr>
</thead>
<tbody>
<tr>
<td>None *</td>
<td>0.468696</td>
<td>14.54569</td>
<td>14.2646</td>
<td>0.0451</td>
</tr>
<tr>
<td>At most 1</td>
<td>0.051833</td>
<td>1.224157</td>
<td>3.841466</td>
<td>0.2685</td>
</tr>
</tbody>
</table>

Max–eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level

hypothesis that a single unit root exists in NII as well as in ΔNII. The result is that the null hypothesis of a unit root in NII is not rejected, while that in ΔNII is rejected. Then, it suggests that both RGDP and NII are characterized by I(1) process. Table A–2 and A–3 present the results of VAR-based cointegration tests developed by Johansen (1991, 1995). The results indicate that the null hypothesis of no cointegration is rejected at 5 percent significance level. The results indicate that RGDP and NII are cointegrated and have a long run equilibrium relationship.

References

Carter, A. P. (1970) Structural change in the American Economy, Harvard University Press,
Cambridge, USA.


