Abstract

The current analysis model calculates the transition probability matrix based on the results of market research of consumer behavior. This analysis is introduced to solve the problem of estimating consumer behavior and to forecast the consumer behavior among three trade areas. Furthermore, consideration based on the results of market research is given to this theoretical model. The significant features of the analysis model are the transition probability matrix among trade areas based on a sample of inflow-outflow investigation for consumer behavior, and the attraction index of trade area introduced from the transition probability matrix.

1. Introduction

We propose the analysis theory based on market research to forecast consumer behavior among regions. Consumer behavior is dependent on the attraction of retail supply as well as the real distance between the physically separated demand and supply. It has been argued by several authors that consumer behavior can be related to physical position between consumer demand and retail supply (Reily, 1929; Converse, 1949; Kotlar, 1971). Also, a network model for consumer behavior has been studied from an analogy of classical transport in resistor networks (White and Ellis, 1971). Much significant study has been conducted in this field. Among many other advances, Huff has conducted many practical experiments on consumer behavior and developed a detailed analysis theory (Huff, 1962).
There is, however, no analysis theory for consumer behavior based on market research. In this paper, we propose a probability analysis model that uses information obtained from market research. The conventional studies that have used only physical factors could not realistically forecast dizzyly changing consumer behavior. We must carefully inspect and analyze the structure of consumer behavior, and must reflect the information obtained through market research in the consumer behavior model. This analysis does not depict the relationship between consumers and retail stores, but it does analyze the relationship between a region with stores and its competing regions.

In Section 2 we describe the method of constructing the fundamental model which analyzes consumer behavior of trade areas in a city. Next we present in theory the forecast of consumer behavior when the number of stores within a trade area has changed. In Section 3, we propose a numerical example to analyze consumer behavior among three trade areas. Furthermore, we forecast consumer behavior among three trade areas when the number of stores has increased. Next we comment on consumer behavior among three trade areas when a large-scale retail store has made inroads into the neighborhood adjacent to the trade areas. In Section 4, we show that the analysis theory can predict consumer behavior among four regions in the Utsunomiya industrial zone.

This study is suggested from the fundamental concept of the Input-Output Table (Leontief, 1986; Miller and Blair, 1985) for consumer behavior between regions, including stores. We feel that the present analysis is very promising for the study of consumer behavior.

2. Input-Output Marketing Analysis Table

2.1. Construction of Fundamental Model

In order to construct the model, the following assumptions must be introduced as elements of the consumer input-output analysis table.

1) There is a one-to-one corresponding relationship between each consumer and each trade area. That is, one consumer belongs to each trade area.

2) The number of inflow and outflow consumers from a trade area is proportional to the number of consumers in that trade area.

3) The sum total of each trade area’s number of individual inflow and outflow consumers is equal to the number of consumers occurring in those trade areas at the same time. In other words, there are no interactions between the consumer behavior of each trade area.

When the city’s economic consumption sphere is constructed of trade areas \( t_1, \ldots, t_n \), we consider the inflow and outflow of consumers among these trade areas. If there is a number \( x_{ij} \) of consumers shopping from trade area \( t_i \) to trade area \( t_j \), from the linearity assumption, the proportion coefficient \( a_{ij} \) is obtained:
where \( C \) indicates consumers belonging to trade area \( t_i \). The proportion coefficient \( a_{ij} \) shows the rate of consumers shopping from trade area \( t_i \) to trade area \( t_j \). For this kind of transition probability \( a_{ij} \), an arrangement of all trade areas \( t_j \)'s transition coefficient for all the trade area \( t_i \) is called a transition probability matrix:

\[
A = \begin{pmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix} \quad \text{transition probability matrix}
\]  

(2)

Each column of this transition matrix \( A \) indicates the probability of each trade area's inflow of consumers. The numerical value indicates the degree of ability to attract consumers from trade area \( t_j \) to trade area \( t_i \). The balance equation of consumers is expressed as,

\[
\begin{align*}
x_{11} + x_{12} + \cdots + x_{1n} &= C_1 \\
x_{21} + x_{22} + \cdots + x_{2n} &= C_2 \\
\vdots & \vdots \\
x_{n1} + x_{n2} + \cdots + x_{nn} &= C_n
\end{align*}
\]

(3)

or, when using the coefficient of transition probability,

\[
\begin{align*}
a_{11} \cdot C_1 + a_{12} \cdot C_1 + \cdots + a_{1n} \cdot C_1 &= C_1 \\
a_{21} \cdot C_2 + a_{22} \cdot C_2 + \cdots + a_{2n} \cdot C_2 &= C_2 \\
\vdots & \vdots \\
a_{n1} \cdot C_n + a_{n2} \cdot C_n + \cdots + a_{nn} \cdot C_n &= C_n
\end{align*}
\]

(4)

is given. The number of customers \( Y_j \) shopping in trade area \( t_j \) is,
When this is expressed via the transition probability coefficient, it becomes,

\[
\begin{align*}
  x_{11} + x_{21} + \cdots + x_{n1} &= Y_1 \\
  x_{12} + x_{22} + \cdots + x_{n2} &= Y_2 \\
  \vdots & \quad \vdots \\
  x_{1n} + x_{2n} + \cdots + x_{nn} &= Y_n
\end{align*}
\]  

When this is expressed via the transition probability coefficient, it becomes,

\[
\begin{align*}
  a_{11} \cdot C_1 + a_{21} \cdot C_2 + \cdots + a_{n1} \cdot C_n &= Y_1 \\
  a_{12} \cdot C_1 + a_{22} \cdot C_2 + \cdots + a_{n2} \cdot C_n &= Y_2 \\
  \vdots & \quad \vdots \\
  a_{1n} \cdot C_1 + a_{2n} \cdot C_2 + \cdots + a_{nn} \cdot C_n &= Y_n
\end{align*}
\]  

When this equation (6) is expressed by using the transition matrix \( A \) and the consumer matrix \( C \), the following equation (7) is attained:

\[
Y = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix} = A^t \cdot C = \begin{pmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix} \cdot \begin{pmatrix}
  C_1 \\
  C_2 \\
  \vdots \\
  C_n
\end{pmatrix}
\]  

where \( t \) indicates the transposed matrix. By using the transition matrix in this way, we can forecast the number of consumers shopping from trade area \( t_i \) to trade area \( t_j \).

Also, the consumer behavior among trade areas can be calculated by the following equation:
Table 1: Input-Output Marketing Analysis Table

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Trade areas</th>
<th>Number of outflow consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( l_1 )</td>
<td>( x_{11} )</td>
<td>( C_1 )</td>
</tr>
<tr>
<td></td>
<td>( l_2 )</td>
<td>( x_{12} )</td>
<td></td>
</tr>
<tr>
<td>Trade areas</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td></td>
<td>( l_n )</td>
<td>( x_{1n} )</td>
<td>( C_n )</td>
</tr>
<tr>
<td>Number of inflow consumers</td>
<td>( Y_1 )</td>
<td>( Y_2 )</td>
<td>( \vdots )</td>
</tr>
</tbody>
</table>

\[
X = \begin{pmatrix}
   x_{11} & x_{12} & \cdots & x_{1n} \\
   x_{21} & x_{22} & \cdots & x_{2n} \\
   \vdots & \vdots & \ddots & \vdots \\
   x_{n1} & x_{n2} & \cdots & x_{nn}
\end{pmatrix}

= C^T \cdot A
\]

\[
C = \begin{pmatrix}
   C_1 & 0 & \cdots & 0 \\
   0 & C_2 & \cdots & \vdots \\
   \vdots & \vdots & \ddots & \vdots \\
   0 & \cdots & \cdots & C_n
\end{pmatrix}

= \begin{pmatrix}
   a_{11} \cdot C_1 & a_{12} \cdot C_1 & \cdots & a_{1n} \cdot C_1 \\
   a_{21} \cdot C_2 & a_{22} \cdot C_2 & \cdots & a_{2n} \cdot C_2 \\
   \vdots & \vdots & \ddots & \vdots \\
   a_{n1} \cdot C_n & a_{n2} \cdot C_n & \cdots & a_{nn} \cdot C_n
\end{pmatrix}
\]

Matrix \( X \), which contains the element \( x_{il} \), represents the input-output marketing analysis table as shown in Table 1. It is possible to forecast consumer behavior among trade areas or the number of customers accumulated within each trade area. Furthermore, when the number of consumers in each trade area changes, we can forecast consumer behavior among trade areas and the number of customers shopping in each trade area.

2.2. Forecast of Consumer Behavior among Trade Areas

When the number of retail stores within a trade area has changed, the ability to attract consumers into its trade area and the number of consumers shopping in its trade area increase. In the case of information related to the transition matrix being available, if the attraction index of a trade area, that is, the ability to attract consumers to a trade area.

area, is tentatively examined, the transition probability matrix can be created using direct and indirect information.

Presently, if the attraction index of a trade area is defined as the sum total of the transition probability of consumers shopping from trade areas $t_j$ to trade area $t_i$, the attraction index $F(f_j)$ is shown as follows:

$$
F(f_j) \equiv \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \cdot A \cdot \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \quad \text{when } \sum_{j=1}^{n} f_j = n
$$

(9)

If the attraction index $F$ is changed to $F'$, it becomes,

$$
F'(f_j') \equiv \begin{pmatrix} f_1' \\ f_2' \\ \vdots \\ f_n' \end{pmatrix} \quad \text{when } \sum_{j=1}^{n} f_j' = n
$$

(10)

The new transition matrix $A'$ can be found upon using the following recurrent equation with modification diagonal matrices $R$ and $S$:

$$
A_{(h+1)} = A_{(h)} \cdot S_{(h)}, \quad A_{(h+2)} = R_{(h+1)} \cdot A_{(h+1)}, \quad (h = 0, 1, 2, \cdots)
$$

(11)

$$
A_{(0)}, R_{(0)}, S_{(0)}: \text{ initial value}
$$

when

$$
A_{(0)} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}
$$

$$
R_{(h)} = \begin{pmatrix}
1 & \vdots & \vdots & \vdots \\
(h) \sum_{j=1}^{n} a_{1j} & 1 & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
(h) \sum_{j=1}^{n} a_{nj} & \vdots & \vdots & 1 \\
\end{pmatrix}, \quad S_{(h)} = \begin{pmatrix}
\frac{f_1'}{n} & \vdots & \vdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\frac{f_n'}{n} & \vdots & \vdots & 0 \\
\end{pmatrix}
$$
The most recently calculated transition matrix $A'$ and the consumer matrix $C$ forecast the number of customers shopping in each trade area by using the following equation:

$$Y' = A' \cdot C = \begin{pmatrix}
    a'_{11} & a'_{12} & \cdots & a'_{1n} \\
    a'_{21} & a'_{22} & \cdots & a'_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a'_{n1} & a'_{n2} & \cdots & a'_{nn}
\end{pmatrix}
\begin{pmatrix}
    C_1 \\
    C_2 \\
    \vdots \\
    C_n
\end{pmatrix}$$

(12)

where $t$ indicates the transposed matrix. The most recently calculated new transition matrix $A'$ finds the input-output marketing analysis table as shown in Table 2, which forecasts consumer behavior by the following equation:

$$X' = C^+ \cdot A' = \begin{pmatrix}
    x'_{11} & x'_{12} & \cdots & x'_{1n} \\
    x'_{21} & x'_{22} & \cdots & x'_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    x'_{n1} & x'_{n2} & \cdots & x'_{nn}
\end{pmatrix}
\begin{pmatrix}
    a'_{11} & a'_{12} & \cdots & a'_{1n} \\
    a'_{21} & a'_{22} & \cdots & a'_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a'_{n1} & a'_{n2} & \cdots & a'_{nn}
\end{pmatrix}
\begin{pmatrix}
    C_1 \\
    C_2 \\
    \vdots \\
    C_n
\end{pmatrix}$$

(13)
Table 2: Input-Output Marketing Analysis Table

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Trade areas</th>
<th>Number of outflow consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$t_1$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>Trade areas</td>
<td>$t_1$</td>
<td>$x'_{11}$</td>
<td>$x'_{12}$</td>
</tr>
<tr>
<td></td>
<td>$t_2$</td>
<td>$x'_{21}$</td>
<td>$x'_{22}$</td>
</tr>
<tr>
<td></td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>Trade areas</td>
<td>$t_n$</td>
<td>$x'_{n1}$</td>
<td>$x'_{n2}$</td>
</tr>
</tbody>
</table>

| Number of inflow consumers | $Y_1'$ | $Y_2'$ | $\cdots$ | $Y_n'$ |

Figure 1: Structure of Three Trade Areas

3. Numerical Example for Three Trade Areas

3.1. Consumer Behavior among Three Trade Areas

We explain how consumer behavior is forecasted by using the numerical example of the simplest case of three trade areas. There are three trade areas, that is, trade areas $t_1$, $t_2$, and $t_3$, as shown in Figure 1. The number of consumers belonging to these trade areas is 60 units, 55 units, and 45 units, respectively. However, one unit can have up to 10,000 people or number of households.

According to the market survey for a definite period of time, the number of
consumers shopping from trade area $t_1$ to trade area $t_1$ was 105. The numbers of consumers shopping from trade area $t_1$ to trade area $t_2$ and from trade area $t_1$ to trade area $t_3$ were 45 and 30, respectively. At the same time, we can also find the number of consumers shopping from trade area $t_2$ and trade area $t_3$, as shown in Table 3. The coefficient of transition probability $a_y$ is each element $e_y$ of the consumer inflow-outflow table shown in Table 3 divided by the number of outflow consumers $E_y$, as follows:

$$
A = \begin{pmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{pmatrix}
= \begin{pmatrix}
  e_{11} & e_{12} & e_{13} \\
  e_{21} & e_{22} & e_{23} \\
  e_{31} & e_{32} & e_{33}
\end{pmatrix}
= \begin{pmatrix}
  0.5833 & 0.2500 & 0.1667 \\
  0.2727 & 0.4545 & 0.2727 \\
  0.3333 & 0.2222 & 0.4444
\end{pmatrix} \quad (14)
$$

From the transition matrix $A$ and the consumer matrix $C$, we can find the number of customers shopping in trade areas $t_1$, $t_2$ and $t_3$, as found in the following equation:

$$
Y = \begin{pmatrix}
  Y_1 \\
  Y_2 \\
  Y_3
\end{pmatrix} = tA \cdot C = \begin{pmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
  C_1 \\
  C_2 \\
  C_3
\end{pmatrix} = \begin{pmatrix}
  65 \\
  50 \\
  45
\end{pmatrix} \quad (15)
$$

where $t$ indicates the transposed matrix. For that reason, it is forecasted that the number of customers shopping in trade area $t_1$ is 65 units, in trade area $t_2$ is 50 units, and in trade area $t_3$ is 45 units.
Table 4: Consumer Input-Output Marketing Analysis Table

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Trade areas</th>
<th>Number of outflow consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t₁</td>
<td>t₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C₁</td>
<td>C₂</td>
</tr>
<tr>
<td>t₁</td>
<td>x₁₁</td>
<td>x₁₂</td>
<td>x₁₃</td>
</tr>
<tr>
<td>(35)</td>
<td>(15)</td>
<td>(10)</td>
<td></td>
</tr>
<tr>
<td>Trade areas t₂</td>
<td>x₂₁</td>
<td>x₂₂</td>
<td>x₂₃</td>
</tr>
<tr>
<td>(15)</td>
<td>(25)</td>
<td>(15)</td>
<td></td>
</tr>
<tr>
<td>t₃</td>
<td>x₃₁</td>
<td>x₃₂</td>
<td>x₃₃</td>
</tr>
<tr>
<td>(15)</td>
<td>(10)</td>
<td>(20)</td>
<td></td>
</tr>
<tr>
<td>Number of inflow consumers</td>
<td>Y₁</td>
<td>Y₂</td>
<td>Y₃</td>
</tr>
<tr>
<td>(65)</td>
<td>(50)</td>
<td>(45)</td>
<td></td>
</tr>
</tbody>
</table>

and in trade area t₃, is 45 units. The following equation (16) can be used to forecast these consumer behaviors among the three trade areas.

\[ X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = C^* \cdot A \]

\[ = \begin{bmatrix} C₁ & 0 & 0 \\ 0 & C₂ & 0 \\ 0 & 0 & C₃ \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 35 & 15 & 10 \\ 15 & 25 & 15 \\ 15 & 10 & 20 \end{bmatrix} \]  \quad (16)

Matrix \( X \), which has the elements \( x_{ij} \), arrives to the input-output marketing analysis table as shown in Table 4. In this way the transition matrix, which introduces the ability to attract consumers to a trade area, can be used to calculate these consumer behaviors between trade areas.

### 3.2. Consumer Behavior with Re-equipment of Shopping Center

When the shopping center within trade area \( t₁ \) has re-equipped, or the number of stores within trade area \( t₃ \) has changed, we are able to forecast the changes in consumer behavior among the three trade areas. The attraction matrix of ability to attract consumers is based on the definition in equation (9):

\[ F(f_j) = \begin{bmatrix} f₁ \\ f₂ \\ f₃ \end{bmatrix} = A \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.1894 \\ 0.9268 \\ 0.8838 \end{bmatrix} \]  \quad (17)
where $F(f_j)$ indicates a relative attraction index of each trade area. When the shopping center within trade area $t_i$ has been re-equipped, and the attraction index of trade area $t_i$ is changed from 0.8838 to 1.2, a new attraction index $F'(f_j)$ is calculated as follows:

$$F'(f_j) = \begin{pmatrix} f'_1 \\ f'_2 \\ f'_3 \end{pmatrix} = \begin{pmatrix} 1.0117 \\ 0.7883 \\ 1.2 \end{pmatrix} ; \quad \text{when } \sum_{j=1}^{3} f'_j = 3$$ (18)

The new transition matrix $A'$ can be sought by using the following recurrent equation:

$$A_{(h+1)} = A_{(h)} \cdot S_{(h)}, \quad A_{(h+2)} = R_{(h+1)} \cdot A_{(h+1)} \quad (h = 0,1,2,\cdots)$$ (19)

In this way, the transition matrix $A'$ can be obtained by modification matrices $R$ and $S$.

$$A' = \begin{pmatrix} 0.5243 & 0.2271 & 0.2486 \\ 0.2302 & 0.3878 & 0.3820 \\ 0.2573 & 0.1734 & 0.5694 \end{pmatrix}$$ (20)
Table 5: Consumer Input-Output Marketing Analysis Table

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Trade areas</th>
<th>Number of outflow consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_1$</td>
<td>$t_2$</td>
<td>$t_3$</td>
</tr>
<tr>
<td></td>
<td>$x'_{11}$</td>
<td>$x'_{12}$</td>
<td>$x'_{13}$</td>
</tr>
<tr>
<td></td>
<td>(31)</td>
<td>(14)</td>
<td>(15)</td>
</tr>
<tr>
<td>Trade areas</td>
<td>$t_2$</td>
<td>$t_3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x'_{21}$</td>
<td>$x'_{22}$</td>
<td>$x'_{23}$</td>
</tr>
<tr>
<td></td>
<td>(13)</td>
<td>(21)</td>
<td>(21)</td>
</tr>
<tr>
<td></td>
<td>$x'_{31}$</td>
<td>$x'_{32}$</td>
<td>$x'_{33}$</td>
</tr>
<tr>
<td></td>
<td>(11)</td>
<td>(8)</td>
<td>(26)</td>
</tr>
<tr>
<td>Number of inflow consumers</td>
<td>$Y'_1$</td>
<td>$Y'_2$</td>
<td>$Y'_3$</td>
</tr>
<tr>
<td></td>
<td>(55)</td>
<td>(43)</td>
<td>(52)</td>
</tr>
</tbody>
</table>

Figure 3: Consumer Input-Output Diagram

In accordance with the most recently calculated transition matrix $A'$, the input-output marketing analysis table, which forecasts consumer behavior, can be found as follows:

$$X' = \begin{bmatrix} x'_{11} & x'_{12} & x'_{13} \\ x'_{21} & x'_{22} & x'_{23} \\ x'_{31} & x'_{32} & x'_{33} \end{bmatrix} = C' \cdot A' = \begin{bmatrix} 31 & 14 & 15 \\ 13 & 21 & 21 \\ 11 & 8 & 26 \end{bmatrix}$$ (21)
Tables 4 and 5 and Figures 2 and 3 show that because the attraction index of trade area \( t_3 \) had increased, the consumers in trade area \( t_3 \) had not shopped in trade areas \( t_1 \) and \( t_2 \) but only within trade area \( t_3 \). On the other hand, the number of consumers from outside regions shopping in trade area \( t_3 \) increased. The decrease of consumers in trade areas \( t_1 \) and \( t_2 \) is an extremely interesting phenomenon. We can forecast consumer behavior among trade areas by using this technique.

### 3.3. Influence of Inroads of a Large-scale Retail Store

In this section, we presuppose that a large-scale retail store is scheduled to make inroads into the neighborhood adjacent to trade areas \( t_1 \) and \( t_3 \). Because the characteristics and primary marketing factors of a large-scale retail store are equivalent to those of a trade area, the large-scale retail store forms a new trade area. Even if the information relating to the inroads of a retail store is scarce, we can obtain the transition matrix which forecasts the changes in consumer behavior among three trade areas and a fourth trade area from existing consumer behavior within the three trade areas.

Suppose that a large-scale retail store has made inroads into the neighborhood adjacent to trade areas \( t_1 \) and \( t_3 \) and now forms new trade area \( t_4 \) as shown in Figure 4. For the sake of this exercise, the number of consumers belonging to trade area \( t_4 \) consist of 20 units which have transferred from trade area \( t_1 \) and 15 units which have transferred from trade area \( t_3 \). There is a great variety in the technique of how consumers move, but they are not discussed in this paper.

In the previous section, the transition matrix \( A \) of the present three trade areas has
been already found. We estimate the initial value of the transition matrix of the four trade areas in accordance with the foundation of the new trade area $t_4$. Furthermore, the attraction index of the previous three trade $t_1$, $t_2$ and $t_3$ is as per below. Namely,

$$
F(f) = \begin{pmatrix}
  f_1 \\
  f_2 \\
  f_3 \\
  f_4
\end{pmatrix} = A \cdot \begin{pmatrix}
  1 \\
  1 \\
  1
\end{pmatrix} = \begin{pmatrix}
  1.1894 \\
  0.9268 \\
  0.8838
\end{pmatrix}
$$

(22)

Here, because the only information related to the trade area $t_4$ is the relative attraction index, the initial transition probability of trade area $t_4$ is the average of that of trade areas $t_1$, $t_2$ and $t_3$. However, if the relative information is available, the weighting average is preferred.

$$
A_{(0)} = \begin{pmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} \\
  a_{21} & a_{22} & a_{23} & a_{24} \\
  a_{31} & a_{32} & a_{33} & a_{34} \\
  a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
$$

\[
= \begin{pmatrix}
  a_{11} & a_{12} & a_{13} & \frac{a_{12} + a_{13}}{2} \\
  a_{21} & a_{22} & a_{23} & \frac{a_{21} + a_{23}}{2} \\
  a_{31} & a_{32} & a_{33} & \frac{a_{31} + a_{32}}{2} \\
  \frac{a_{21} + a_{31}}{2} & \frac{a_{12} + a_{32}}{2} & \frac{a_{13} + a_{33}}{2} & \frac{a_{11} + a_{22} + a_{33}}{3}
\end{pmatrix}
\]

(23)

Supposing the large-scale retail store makes inroads and trade area $t_4$ is formed, and the attraction index $f_i$ of its trade area is 1.2, the attraction index $F'(f)$ is calculated as follows:
The new transition matrix $A'$ is obtained from the initial transition matrix $A_0$, the new attraction index $F'(f'_i)$, and the recurrent equation (11).

$$
A' = \begin{pmatrix}
0.4554 & 0.1975 & 0.1331 & 0.2140 \\
0.1990 & 0.3357 & 0.2035 & 0.2619 \\
0.2418 & 0.1632 & 0.3298 & 0.2652 \\
0.2139 & 0.1687 & 0.1586 & 0.4589
\end{pmatrix}
$$

(25)

Newly calculated transition matrix $A'$ introduces the consumer input-output marketing analysis table and examines the influence of the existing three trade areas in which the large-scale retail store has made inroads.

$$
X' = \begin{pmatrix}
x'_{11} & x'_{12} & x'_{13} & x'_{14} \\
x'_{21} & x'_{22} & x'_{23} & x'_{24} \\
x'_{31} & x'_{32} & x'_{33} & x'_{34} \\
x'_{41} & x'_{42} & x'_{43} & x'_{44}
\end{pmatrix} = C'.A' = \begin{pmatrix}
18 & 8 & 5 & 9 \\
11 & 19 & 11 & 14 \\
7 & 5 & 10 & 8 \\
7 & 6 & 6 & 16
\end{pmatrix}
$$

(26)

The consumer input-output marketing analysis table shown in Table 6 can be used to forecast changes of consumer behavior occurring within each trade area. When a large-scale retail store has made inroads into the neighborhood adjacent to trade areas $t_1$ and $t_3$, the consumer behavior among each trade area can be forecasted by using this table. Next we will show how diagrams use the consumer behavior before and after inroads into the trade area. Consumer behavior before and after inroads of a large-scale retail store are shown in the following diagram. Although trade area $t_2$ is positioned relatively far from trade area $t_4$, many consumers are still shopping in trade area $t_4$. Similarly, consumers shopping from trade areas $t_1$ and $t_3$ to trade area $t_2$ prior to inroads of the large-scale retail store have decreased significantly, and consumers shopping in trade area $t_4$ have increased. We cannot conclude that the influence of inroads of the large-scale retail store is weak just because it is far away. This influence
Table 6: Consumer Input-Output Marketing Analysis Table

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Trade areas</th>
<th>Number of outflow consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_1$</td>
<td>$t_2$</td>
<td>$t_3$</td>
</tr>
<tr>
<td></td>
<td>$x'_{11}$</td>
<td>$x'_{12}$</td>
<td>$x'_{13}$</td>
</tr>
<tr>
<td>Trade areas</td>
<td>$t_2$</td>
<td>$t_3$</td>
<td>$t_4$</td>
</tr>
<tr>
<td></td>
<td>$x'_{21}$</td>
<td>$x'_{22}$</td>
<td>$x'_{23}$</td>
</tr>
<tr>
<td></td>
<td>$x'_{31}$</td>
<td>$x'_{32}$</td>
<td>$x'_{33}$</td>
</tr>
<tr>
<td></td>
<td>$t_4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x'_{41}$</td>
<td>$x'_{42}$</td>
<td>$x'_{43}$</td>
</tr>
<tr>
<td>Number of inflow consumers</td>
<td>$Y_1^1$</td>
<td>$Y_3^1$</td>
<td>$Y_3^1$</td>
</tr>
<tr>
<td></td>
<td>(43)</td>
<td>(38)</td>
<td>(26)</td>
</tr>
</tbody>
</table>

Figure 5: Consumer Inflow-Outflow Diagram

of the large-scale retail store first spreads to the most adjacent trade area and then spreads to the next most adjacent trade area. In this way, we can understand the process by which this influence extends to one trade area after another.

When the large-scale retail store makes inroads into the neighborhood adjacent to trade areas $t_1$ and $t_3$, in the same way as other trade areas new trade area $t_4$ is formed according to retail characteristics and marketing factors of that large-scale retail
store. From results of this analysis we understand that the ability to attract consumers from other trade areas is extremely large. In this case of large-scale retail store inroads, this technique can be extended to quantitatively forecast the influence of changes in consumer behavior within each trade area.

4. Empirical Test for Input-Output Marketing Analysis

When the shopping center has re-equipped, or the number of stores has changed, we demonstrate how consumer behavior within the Utsunomiya industrial zone in 1991 is forecasted from consumer behavior in 1988. The 13 administrative districts in the Utsunomiya industrial zone are shown in Figure 6.

In Table 7 we present the inflow and outflow table based on the market survey for 1988 that has been aggregated into four regions (Tochigi prefectural office, 1988a).
Their regions are Utsunomiya (Utsunomiya), Kanuma (Kanuma, Awano, and Mibu), Mooka (Mooka, Ishibashi, Kaminokawa, and Ninomiya), and Kawachi (Kamikawachi, Kawachi, Uziie, Takanezawa, and Haga).

The most right-hand column is the number of households drawn as a random sample from residents, and the next most right-hand column is the total number of ballots used within this zone. They have 10 ballots and can give their ballots to some regions in which they shop.

The transition probability matrix $A$ is obtained from the row element of the consumer inflow-outflow table shown in Table 7 divided by the number of ballots.

\[
A = \begin{pmatrix}
1.0000 & 0 & 0 & 0 \\
0.2594 & 0.7359 & 0.0047 & 0 \\
0.1458 & 0.0092 & 0.8450 & 0 \\
0.5188 & 0 & 0.0517 & 0.4295
\end{pmatrix}
\] (27)

Matrix $A$, which has the elements of transition probability, arrives to the input-output marketing analysis table as shown in Table 8. In this way, the transition matrix can be used to calculate these consumer behaviors within the industrial zone with four regions.

In the same way, market research has been also assembled for 1991 as shown in Table 9 (Tochigi prefectural office, 1991a).

When the shopping center has re-equipped, or the number of stores has changed, we are able to forecast the changes in consumer behavior within the Utsunomiya industrial zone with four regions. The attraction matrix $F(f_j)$ of ability to attract consumers is based on the definition in equation (9):

\[
F(f_j) = \begin{pmatrix}
 f_1 \\
f_2 \\
f_3 \\
f_4
\end{pmatrix} = \begin{pmatrix}
 1.9240 \\
0.7451 \\
0.9014 \\
0.4295
\end{pmatrix}
\] (28)

In Table 10 we present the floor space of retail stores for 1988 and 1991 that has been aggregated to four regions (Tochigi prefectural office, 1988b, 1991b).

Now we introduce an assumption of the gravity model, such as Huff’s model, that a region’s ability to attract consumers is in proportion to total floor space of retail stores existing in the region. When the number of stores has been increased as shown in Table 10, the new attraction index $F'(f')$ is calculated from the rate of increase as follows:
Table 8: Consumer Input-Output Marketing Analysis Table for 1988

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Utsunomiya industrial zone</th>
<th>Number of households</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t₁</td>
<td>t₂</td>
<td>t₃</td>
</tr>
<tr>
<td>Utsunomiya</td>
<td>t₁</td>
<td>130,997</td>
<td></td>
</tr>
<tr>
<td>Kanuma</td>
<td>t₂</td>
<td>8,963</td>
<td>25,428</td>
</tr>
<tr>
<td>Mooka</td>
<td>t₃</td>
<td>4,423</td>
<td>279</td>
</tr>
<tr>
<td>Kawachi</td>
<td>t₄</td>
<td>12,984</td>
<td>1,294</td>
</tr>
<tr>
<td>Number of consumers</td>
<td>157,347</td>
<td>25,707</td>
<td>27,092</td>
</tr>
</tbody>
</table>

Table 9: Consumer Input-Output Marketing Analysis Table for 1991

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Utsunomiya industrial zone</th>
<th>Number of households</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t₁</td>
<td>t₂</td>
<td>t₃</td>
</tr>
<tr>
<td>Utsunomiya</td>
<td>t₁</td>
<td>143,570</td>
<td>884</td>
</tr>
<tr>
<td>Kanuma</td>
<td>t₂</td>
<td>6,094</td>
<td>31,222</td>
</tr>
<tr>
<td>Mooka</td>
<td>t₃</td>
<td>4,618</td>
<td>559</td>
</tr>
<tr>
<td>Kawachi</td>
<td>t₄</td>
<td>12,363</td>
<td>15</td>
</tr>
<tr>
<td>Number of consumers</td>
<td>166,645</td>
<td>321,680</td>
<td>30,943</td>
</tr>
</tbody>
</table>

Table 10: Floor Space of Retail Stores

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Floor space (m²)</th>
<th>Rate of increase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1988</td>
<td>1991</td>
</tr>
<tr>
<td>Utsunomiya</td>
<td>t₁</td>
<td>434,807</td>
</tr>
<tr>
<td>Kanuma</td>
<td>t₂</td>
<td>122,634</td>
</tr>
<tr>
<td>Mooka</td>
<td>t₃</td>
<td>109,652</td>
</tr>
<tr>
<td>Kawachi</td>
<td>t₄</td>
<td>69,217</td>
</tr>
</tbody>
</table>

\[
F'(f_j') = \begin{pmatrix}
    f'_1 \\
    f'_2 \\
    f'_3 \\
    f'_4
\end{pmatrix} = \begin{pmatrix}
    2.0072 \\
    0.7369 \\
    0.8692 \\
    0.3867
\end{pmatrix}
\] (29)

The new transition matrix \( A' \) can be sought from equations (27), (29), and the recurrent equation (11), namely,
Table 11: Consumer Input-Output Marketing Analysis Table

<table>
<thead>
<tr>
<th>Inflow</th>
<th>Outflow</th>
<th>Utsunomiya industrial zone</th>
<th>Number of households</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$t_1$</td>
<td>$t_2$</td>
</tr>
<tr>
<td>Utsunomiya</td>
<td>$t_1$</td>
<td>144,749</td>
<td>0</td>
</tr>
<tr>
<td>Kanuma</td>
<td>$t_2$</td>
<td>10,109</td>
<td>27,269</td>
</tr>
<tr>
<td>Mooka</td>
<td>$t_3$</td>
<td>5,924</td>
<td>356</td>
</tr>
<tr>
<td>Kawachi</td>
<td>$t_4$</td>
<td>16,053</td>
<td>1,320</td>
</tr>
<tr>
<td>Number of consumers</td>
<td>176,835</td>
<td>27,625</td>
<td>29,812</td>
</tr>
</tbody>
</table>

\[
A' = \begin{pmatrix}
1.0000 & 0 & 0 & 0 \\
0.2694 & 0.7266 & 0.0040 & 0 \\
0.1711 & 0.0103 & 0.8186 & 0 \\
0.5667 & 0.0466 & 0.3867 & 0
\end{pmatrix} \tag{30}
\]

In accordance with the most recently calculated transition matrix $A'$, the input-output marketing analysis table, which forecasts consumer behavior, can be found in Table 11.

The results shown in Tables 9 and 11 are sufficient to show the excellent agreement of the empirical data with the numerical data calculated by the present model. Thus, present consumer behavior as well as the floor space of retail stores and the distance between regions are of considerable importance in determining future consumer behavior between regions.

5. Conclusions

We have proposed the probability analysis model, the input-output marketing analysis table, as a means of analyzing consumer behavior. We have also considered in theory the fundamental way of thinking behind the closed-system analysis and the three examples of the region with three trade areas, and have applied this analytical model to predict consumer behavior in the Utsunomiya industrial zone.

Previous analyses have merely forecasted consumer behavior among trade areas considering only physical distance. The present analysis calculates the transition probability matrix based on results of market research, and formulates the input-output marketing analysis table from that matrix. Defining the relative attraction index of a trade area, and depending on the relationship among trade areas, we were able to forecast the consumer input and output behavior between each trade area.

Finally, the present analysis theory based upon the input-output marketing analysis table can be widely applied to predicting other economic transport phenomena, such as the transport of goods and the transmission of information among regions. The theoretical analysis based on the open system and the numerical example for consumer
behavior among a city, including three trade areas and its competing two cities, shall be presented in a next paper.

References


